Exponential-Growth Bias and Lifecycle Consumption*

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Abstract

Exponential-growth bias (EGB) is the tendency for individuals to partially neglect compounding of exponential growth. We develop a model wherein biased agents misperceive the budget constraint, and derive conditions for overconsumption and dynamic inconsistency. We confirm our predictions in a simulated lifecycle-consumption task. We construct an incentivized measure of EGB in a US-representative population and find substantial bias, with approximately one-third of subjects estimated as the fully-biased type. The magnitude of the bias is negatively associated with asset accumulation, and robust to a simple graphical intervention. Finally, we show laboratory subjects are largely unaware of their bias and undervalue assistance.

Keywords: exponential-growth bias, dynamic inconsistency, personal finance, overconsumption, lifecycle consumption, overconfidence

JEL: D03, D11, D12, D14, D18, D91, E21

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1 Introduction

Virtually all intertemporal financial decisions involve real or nominal values that change exponentially over time. Proper computation of exponential functions is thus at the heart of many economic decisions such as lifecycle consumption and portfolio choice. Many people, however, exhibit a strong systematic bias towards linear growth when estimating such functions. This can have large positive and normative consequences. Moreover, an economist who does not account for this bias may misinterpret observed behavior, such as the pervasive borrowing through payday loans with APRs exceeding 3000%\(^1\) as well as the apparently low retirement savings of many households.\(^2\)

This paper suggests that systematically biased financial errors may be an important missing factor in lifecycle consumption puzzles, and must be considered alongside other explanations such as time-inconsistent preferences, or unobserved substitution to home production.

We construct a model of exponential-growth bias (EGB), the psychological tendency for individuals to underestimate exponential growth due to the neglect of compounding, and test the novel predictions of the model in both lab and artifactual field experiments. EGB leads the agent to make two fundamental errors regarding his intertemporal budget constraint. First, the agent misperceives the relative prices of consumption over time. With positive interest rates this causes the agent to overestimate the price of future consumption relative to present consumption. We call this the \textit{price effect of exponential-growth bias}, and it combines the standard income and substitution effects on consumption choices (albeit over a misperception rather than a real price change).

Second, the consumer misperceives the value of his income over time. With positive interest rates this causes the consumer to overestimate the value of future income. We call this the \textit{wealth effect of exponential-growth bias}, as it operates through a perceived wealth effect on future earnings.\(^3\) We embed this portable model in a lifecycle-consumption environment, yielding novel predictions that

\(^1\) The U.S. payday loan industry is so successful that brick-and-mortar payday loan locations exceed the number of McDonalds and Starbucks combined (Skiba and Tobacman, 2011).

\(^2\) While the adequacy of retirement savings is the subject of ongoing debate, Munnell et al. (2006) argue that more than 40% of U.S. households are saving insufficiently to maintain their standard of living into retirement. Poterba et al. (2011) find that the 30th percentile household age 65–69 had total investment savings (financial assets plus personal retirement account) of $5,500 in 2008, while the median had total investment savings of only $52,000.

\(^3\) We treat income as exogenous to match our experimental design and to investigate purely financial decisions; endogenizing income does not qualitatively change the predictions of the model.
diverge dramatically from classical ones.

While there are folk stories illustrating people’s underestimation of exponential growth going back millennia,\(^4\) to our knowledge, Wagenaar and Sagaria (1975) conducted the first published experiment demonstrating this phenomenon in the psychology literature. Subsequent studies found the same pattern of underestimation (Wagenaar and Timmers, 1979; Keren, 1983; Benzion, Granot and Yagil, 1992; Almenberg and Gerdes, 2012). Wagenaar and Sagaria (1975) wrote an early model of EGB that used two parameters in which an exponential function of the form \(x(t) = a e^{ct}\) is perceived as \(\hat{x}(t) = a e^{\alpha c^t}\). Stango and Zinman (2009) first introduced EGB to the economics literature and extended the Wagenaar-Sagaria model to environments where people face loans with periodic payments. They show that EGB causes people to underestimate the future value of savings and the costs associated with borrowing. Using an interest-rate question as a proxy for EGB on the 1977 and 1983 Survey of Consumer Finances, they find that errors on the question are correlated with a number of important economic outcomes. Those with larger errors have higher short-term debt to income ratios, lower stock ownership as a percentage of portfolios, lower savings rates, lower net worth, and no difference in long-term debt to income ratios, all as predicted by their model.

Our first contribution is the development of a model of EGB that is both empirically accurate and portable. The Wagenaar-Sagaria model captures some basic implications of underestimation, but has several undesirable properties that we improve upon. First, when taken literally, it implies that a biased agent will underestimate exponential growth even after one period when interest has not yet compounded, whereas we find that three quarters of our sample get this exactly correct. Second, the model does not nest full neglect of compounding (misperceiving compound interest as simple interest), which we observe in about one third of our sample. Third, the model predicts that a biased individual will overestimate the value of a depreciating asset, whereas we find evidence of the opposite. Fourth, this model would not predict that an agent, when estimating the constant-rate equivalent of a fluctuating return, will tend to be biased toward the arithmetic mean – not realizing, for example, that they are left strictly worse off by a 10% gain followed by a 10%  

\(^4\)According to legend, the ruler of an Indian kingdom granted the inventor of chess a single boon. The inventor requested a quantity of rice that doubled for every square on the chessboard, starting with a single grain. The ruler quickly accepted the request only to later discover that the sum exceeded the kingdom’s entire store.
loss. The data suggest this is the case. As a result, the biased agent will exhibit \textit{as-if} risk preferences, being insufficiently sensitive to large negative periodic returns. An economist who believed that uncertainty generated the variation in returns would infer that the agent had risk-seeking preferences. We derive weaker conditions on perceptions to define EGB, and develop a simple new single-parameter model to estimate.\footnote{Stango and Zinman (2009) also include a very general model in their online appendix. Our general model is a special case of their general model but puts greater restrictions on what is defined as exponential-growth bias — as opposed to general mathematical errors — thereby focusing directly on the effects of distorted beliefs regarding compounding.}

As our second contribution, we embed our model of EGB in a lifecycle consumption model and explore the behavior of a biased agent. Because agents overestimate the value of future income, shifting income to later periods in a way that preserves lifetime wealth will increase consumption in the present. We derive sufficient conditions under which the consumer will overconsume in the present for any positive income vector and any smooth utility function. While commitment is generally thought to help present-biased agents, it will in fact often exacerbate overconsumption for an EGB agent who locks in his biased consumption plans. Moreover, because the perception of future prices and lifetime wealth changes each period, the agent will behave in a dynamically inconsistent manner that is distinct from the pattern generated by dynamically inconsistent time preference: he will revise his consumption plans upward when he is a net saver and downwards when he is a net borrower.

We believe that present-biased and other dynamically inconsistent preferences play an important role in intertemporal consumption decisions, but that many financial choices are more plausibly explained with the presence of EGB. EGB enables procrastination by decreasing perceived future costs. For example, in order to explain take-up of loans at triple- and quadruple-digit interest rates, consumers with only present bias and not EGB would require calibrationally extreme discount factors. A fully biased agent, conversely, could easily misperceive the costs of debt by orders of magnitude. An additional distinction is that a present-biased agent with access to credit (and no commitment devices) would choose to receive income to maximize his intertemporal budget constraint just like an exponential discounter, while an EGB agent may not.

Our third contribution is a test of these predictions in a laboratory lifecycle-consumption ex-
periment. Although there have been other laboratory lifecycle-consumption experiments (Hey and Dardanoni, 1988; Brown, Chua and Camerer, 2009), with one (Johnson, Kotlikoff and Samuelson, 1987) finding behavior consistent with an underestimation of compounding interest, ours is the first to focus on the manifestations of EGB as a function of lifecycle environments. Subjects make consumption choices in a lifecycle environment with an induced utility function. All of the main comparative static predictions of our model are confirmed. Subjects overconsume early in their lifecycle and many fully exhaust their wealth before reaching the final period; overconsumption is dramatically exacerbated when income is received later in the lifecycle; and with commitment, consumption differs starkly from consumption without commitment due to EGB-induced dynamic inconsistency. We find no evidence for learning during the course of the experiment.

As a fourth contribution we measure the pervasiveness of EGB and test the validity of our model in a representative sample of the population. Subjects answer questions about the relative value of two financial assets and are paid for accuracy. Using our model we estimate the accuracy $\alpha$ by subject and find that about one third of the population is fully-biased with $\alpha = 0$. The median bias is 0.6 and 96% of subjects are estimated to have an $\alpha < 1$ (i.e. underestimate compound growth). This is in spite of the fact that subjects participated online and had access to whatever tools (e.g. financial calculators, help from friends) that they chose to use. Various questions also produce “fingerprinted” EGB responses that are predicted only by our model. Regressing log savings on $\alpha$, we find that it enters positively and significantly while controlling for income, education, age, and other covariates. Moving from full bias to full accuracy is associated with a ceteris paribus 55–90% increase in accumulated assets. This augments the Stango-Zinman finding, that bias is correlated with savings and net worth, by using a direct elicitation of EGB instead of a proxy.

We additionally find that $\alpha$ is uncorrelated with age and education, indicating that the bias does not diminish with experience. The evidence suggests that EGB is pervasive in the population, and an important predictor of savings behaviors. Moreover, we find that a graphical “de-biasing” intervention had no effect on performance. While other experimental de-biasing interventions have met with mixed results (MacKinnon and Wearing, 1991; Eisenstein and Hoch, 2007; McKenzie and Liersch, 2011; Soll et al., 2011; Goda et al., Forthcoming; Song, 2012), ours is unique in assessing
the additional effect on biased perceptions when subjects may already use external resources. As a “back of the envelope” calculation, our estimates imply that the median American suffers a welfare loss equivalent to 2–5% of lifetime wealth as a result of EGB-induced mis-optimization.

The next section presents the model. Section 3 describes the laboratory lifecycle-consumption experiment that tests the novel predictions of the theory. In Section 4 we measure the prevalence of EGB in the population. The paper concludes in Section 5.

2 Theory

2.1 Model

We consider an agent who faces a vector of interest rates \( \vec{i} = <i_0, i_1, \ldots, i_{T-1}> \in \mathbb{R}^T \), and a vector of cash flows (income, contributions, debits, etc.) \( \vec{y} = <y_0, \ldots, y_T> \in \mathbb{R}^T \). Both the cash flows and interest rates are certain and known to the agent. The timing of cash flows, and the unit of time treated as a “period” are given exogenously.\(^6\)

We define \( p(\vec{i}, t) \) to be the agent’s perception of the period-\( T \) value of one dollar invested at time \( t \). For example, the correct perception would be \( p(\vec{i}, t) = \prod_{\tau=t}^{T-1} (1 + i_\tau) \), whereas a perception that ignored all compounding would be \( p(\vec{i}, t) = 1 + \sum_{\tau=t}^{T-1} i_\tau \).

We consider a simple one-parameter model of EGB, using the parameter \( \alpha \) to denote the accuracy of the agent’s perception. For ease of exposition, we will refer to an agent whose perception corresponds to a degree \( \alpha \) of exponential-growth bias as an \( \alpha \)-Eddie. While there are many possible functional forms for intermediate degrees of exponential-growth bias, our simple parameterization gives the perceptions of an \( \alpha \)-Eddie as:

\[
p(i, t; \alpha) = \prod_{s=t}^{T-1} (1 + \alpha i_s) + \sum_{s=t}^{T-1} (1 - \alpha) i_s
\]

(1)

The \( \alpha \)-Eddie model is both convenient and well-behaved mathematically, and also has a coherent

\(^6\)The agent’s perceptions are not neutral with respect to the framing of time. In general, finer divisions (e.g. days rather than years) will lead to more opportunity for compounding and hence a greater mistake on the part of the agent. Such non-neutrality is often a feature of models which deviate from neoclassical rationality with exponential discounting – for example, the importance of determining what constitutes the “present” in hyperbolic discounting. We discuss later the implications of this non-neutrality for firms facing biased consumers.
psychological interpretation. A 1-Eddie correctly perceives the asset growing exponentially, while a 0-Eddie is fully biased and perceives the asset growing linearly according to simple interest. An \( \alpha \)-Eddie’s perception corresponds to what would result if part of the interest were siphoned away into a non-compounding account. That is, if a fraction \( \alpha \) of the interest accumulated in an account that will grow in future periods with the interest rate \( \alpha i \), and the remaining fraction \( 1 - \alpha \) of the accumulated interest were sequestered to a non-growing account (e.g. placed under the mattress). This is, of course, not meant to be taken literally. In Appendix A we present a non-parametric general model of exponential-growth bias, which does not rely on a particular functional form of the perception function.

Eddie’s perception at time \( t \) of the period-\( \tau \) value of a general asset with vector of cash flows \( \vec{y}_t = <y_{t+1}, ..., y_T> \) is given by:

\[
\hat{V}_{t,\tau}(\vec{y}_t, \vec{i}; \alpha) = \left( \sum_{s=t}^{\tau} y_s \prod_{r=s}^{\tau-1} (1 + \alpha i_r) \right) + (1 - \alpha) \sum_{s=t}^{\tau} \left( \sum_{r=s}^{\tau-1} i_r \right) y_s
\]

for all \( \tau \leq T \). Eddie will only misperceive the value of an asset at least two periods into the future since \( p(i, T - 1; \alpha) = 1 + i \), which is correct.

The \( \alpha \)-Eddie model has several strengths. First, it is tractable and relatively easy to estimate the single parameter. It is also sufficiently flexible to encompass complex choices. Moreover, while an \( \alpha \)-Eddie’s perception of the growth of an asset converges in the long run to exponential growth, it is at the lower growth rate \( \alpha i \) rather than the true \( i \).

2.2 Theoretical Results

We now explore a biased agent’s behavior in a simple lifecycle-consumption environment. Suppose an agent has an instantaneous utility function over consumption \( u(c_t) \) that is continuously differentiable, strictly concave, and satisfies the Inada conditions: \( u'(0) = \infty \) and \( \lim_{c_t \to \infty} u'(c_t) = 0 \). The agent is born in period 0 and dies in period \( T > 1 \), and he must choose his consumption in each period subject to an intertemporal budget constraint. In each period he receives a (possibly negative) cash flow \( y_t \) and he may purchase or sell shares of a risk-free asset with period-specific
interest rates \( i_t \geq 0 \) (with the inequality strict for at least two periods).\(^7\) We treat income as exogenous for simplicity and to match our later experimental design, but the model easily allows endogenous sources of income.

The agent discounts future utility exponentially by the discount factor \( \delta \leq 1 \). We use conventional exponential discounting to focus on the effect of EGB and isolate its role in behavior. Alternative discounting models such as quasi-hyperbolic discounting can be straightforwardly incorporated. Moreover, we do not model the agent as applying his EGB to his discount function, as EGB is only relevant when numerical values are shrouded by math. Whereas the value of a future asset is often shrouded by exponential calculations, we assume that all explicit and implicit calculations regarding one’s preferences are fully transparent. In other words, the agent knows his intertemporal utility function.\(^8\)

Thus the agent’s period-0 problem is:

\[
\max_{\hat{c}} \sum_{t=0}^{T} \delta^t u(\hat{c}_t) \tag{3}
\]

subject to the budget constraint written in terms of the period-\( T \) value of money,

\[
\sum_{s=0}^{T} \hat{c}_s \cdot p(\hat{\bar{i}}, s; 1) \leq \sum_{s=0}^{T} y_s \cdot p(\bar{i}, s; 1) \tag{4}
\]

where \( p(\bar{i}, t; 1) = \prod_{j=1}^{T-1} (1 + i_j) \) is the correct interest perception. However, since the agent misperceives exponential growth, he believes that his budget constraint is instead:

\[
\sum_{s=0}^{T} \hat{c}_s \cdot p(\hat{\bar{i}}, s; \alpha) \leq \sum_{s=0}^{T} y_s \cdot p(\bar{i}, s; \alpha) \tag{5}
\]

\(^7\)If the interest rate does not differ from zero for at least two periods, then compounding plays no role in the agent’s optimization and our model of EGB does not come into play. We focus on positive interest rates for reasons of exposition and economic relevance, but the propositions are straightforwardly extended to admit negative rates.

\(^8\)As an analogy, someone who has trouble with trigonometry does not lose their depth perception even though the visual system of the brain uses trigonometric calculations to calculate depth. In the same way, the agent’s inability to estimate exponential growth need not place any restrictions on his preferences. While the exponential discounting form is itself the subject of much debate (Frederick et al., 2002), we use it only as a simple approximation of time preference that allows us to place our emphasis on mistakes about the budget constraint.
Fundamentally, the interest rate gives the price of present consumption in terms of the price of future consumption.

Taking the first-order condition, the agent’s period-0 consumption is defined implicitly by:

\[
\sum_{s=0}^{T} p(\bar{t}, s; \alpha) u^{-1} \left( \frac{p(\bar{t}, s; \alpha) u'(c_0)}{p(\bar{t}, 0; \alpha) \delta s} \right) \equiv \sum_{s=0}^{T} y_s \cdot p(\bar{t}, s; \alpha)
\] (6)

Equation (6) specifies what the agent will do in the current period and what the agent plans to do in future periods but not necessarily what the agent actually does in future periods. The agent must satisfy his true budget constraint in the current period: \( c_0 \leq \sum_{s=0}^{T} y_s \cdot p(\bar{t}, s; 1) \). Whoever lends to the agent will ensure that this condition is satisfied. When it binds we will assume that the agent consumes his full wealth in period 0.

The intuition for the theoretical results of this paper lies in Equation (6). Exponential-growth bias leads to two perceptual errors: one for each side of the equation. On the left-hand side, the agent misperceives the relative prices of consumption over time. On the right-hand side, he misperceives his lifetime wealth.

**Lemma 1 (Under-estimation by period)** Suppose \( 0 \leq \alpha < \alpha' < 1 \), then:

\[
\frac{p(\bar{t}, t; \alpha)}{p(\bar{t}, t+1; \alpha)} \leq \frac{p(\bar{t}, t; \alpha')}{p(\bar{t}, t+1; \alpha')}
\]

The inequality holds strictly if there exist \( j > k > t \) s.t. \( i_j > 0, i_k > 0 \).

Lemma 1 implies that if interest rates are always positive then a biased agent perceives the price of future consumption to be relatively too high, and the price of present consumption relatively too low. We henceforth refer to this as the price effect of exponential-growth bias. As in standard consumer theory, a change in prices (albeit a misperception in this case) leads to an income effect and a substitution effect. Since future prices are perceived to be higher than they actually are ($1 today is perceived to buy less in the future than it actually can), income is perceived to be lower. This force will generally decrease planned consumption in all periods. But since the relative prices of early periods are perceived to be lower than they actually are, this will cause more planned...
consumption in early periods and less planned consumption in later periods. The net change in immediate consumption therefore depends on the elasticity of intertemporal substitution (EIS), which we turn to in Proposition 3.

On the right-hand side of Equation (6) the agent misperceives the future value of his income. If \(\alpha < 1\) then \(p(\tilde{\tau}, t; \alpha)\) will be too low and so he will overestimate the present value of future income. In other words, the agent underestimates his budget when income is received early in life but he overestimates income when he receives it late in life. We henceforth refer to this as the \textit{wealth effect of exponential-growth bias}.\footnote{Because we are treating the cash flows as exogenous, there is no possibility of substitution. Extending the model to endogenize income, for example by including a labor supply decision, would naturally lead to a similar income and substitution effect to those for consumption choices.}

The wealth effect leads to our first result. An agent with EGB will perceive an income stream that delays income as more valuable than an equally valuable income stream that expedites it.\footnote{For example, suppose the interest rate is \(i = 9\%\) and \(T = 10\). The value of $100 in \(t = 0\) is $237 in \(t = 10\). A fully biased agent, however, will perceive the $100 in \(t = 0\) as worth $190 in \(t = 10\). An unbiased decision maker would be indifferent between an income stream in which $100 is received in \(t = 0\) and nothing in all other periods, and a second stream in which $237 is received in \(t = 10\) and nothing in all other periods. The biased agent would gladly choose the latter because he overestimates its present value.}

\textbf{Lemma 2 (Income Deferment)} Given income streams \(\tilde{y}\) and \(\tilde{z}\) and interest rates \(\tilde{\tau}\) such that:

\begin{align*}
(i) & \quad \sum_{s=0}^{T} \left( \Pi^{T}_{j=s}(1 + i_{j}) \right) y_{s} = \sum_{s=0}^{T} \left( \Pi^{T}_{j=s}(1 + i_{j}) \right) z_{s} > 0 \\
(ii) & \quad \sum_{s=0}^{t} \left( \Pi^{t}_{j=s}(1 + i_{j}) \right) y_{s} > \sum_{s=0}^{t} \left( \Pi^{t}_{j=s}(1 + i_{j}) \right) z_{s} \quad \forall t \in \{0, \ldots, T - 2\},
\end{align*}

then \(\tilde{V}_{0,T}(\tilde{y}; \tilde{\tau}; \alpha) < \tilde{V}_{0,T}(\tilde{z}; \tilde{\tau}; \alpha)\) for any \(\alpha < 1\).
Since income received in later periods is overvalued and income received in the present is undervalued, delaying income leads to the misperception that the value of lifetime income is greater than it actually is. The wealth effect thus leads to our first result.

**Proposition 1 (Deferred Income Increases Consumption)** \( \text{Delaying income from period } t < (T - 1) \text{ to } \tau > t \text{ in a manner that keeps (true) lifetime income unchanged will cause an agent with EGB to increase consumption in period } 0. \)

The implication is that when the agent receives compensation stated nominally and received in the future, he will overestimate his budget and overconsume in the present. The larger the delay, the larger the error. For sufficiently large bias and sufficiently high interest rates, a lump-sum payment to the agent late in the lifecycle could even make him worse off.

From Proposition 1, if income is sufficiently delayed, interest rates are sufficiently high, and the agent is sufficiently biased, the agent will overconsume in \( t = 0 \) relative to the optimum for any preferences. This provides the basis for Proposition 2.

**Proposition 2 (Overconsumption From Future Wealth)** \( \text{If an agent receives all of his wealth in the last period } T > 1, \text{ then period-0 consumption is higher for more biased interest perception functions (i.e. increasing in EGB).} \)

Proposition 2 states that any biased agent who receives all his wealth in the last period will overconsume at the beginning of his lifecycle. This is a sufficient condition for overconsumption, though by no means a necessary one. If, for example, income tends to be received very early, the agent may overconsume if the perceived price effect dominates the wealth effect. Thus the price effect alone can also generate another sufficient condition for over-consumption.

**Proposition 3 (Overconsumption)** \( \text{If an agent faces a vector of weakly-positive cash flows } \bar{y} (y_t \geq 0 \forall t, \exists s \text{ s.t. } y_s > 0), \text{ then the period-0 consumption of an agent with EGB is greater than that of an unbiased agent if } -\frac{u'(c)}{u''(c)c} > 1, \text{ i.e. the elasticity of intertemporal substitution (EIS) is greater than one.} \)
Proposition 3 states that as long as the EIS > 1, a biased agent will overconsume at the beginning of his lifecycle.\footnote{This result generalizes the under-saving result of Goda et al. (Forthcoming). Their 2-period model, in which all income is received in period 1, generates overconsumption whenever the EIS > 1.} This of course also implies that a necessary condition for underconsumption is that the elasticity of intertemporal substitution is less than one. We stress this is not a sufficient condition for underconsumption, however, as even if the agent is highly inelastic (i.e. his instantaneous utility function is very concave and therefore EIS is very low), from Proposition 2 he will still overconsume in period 0 if his wealth is received as a lump-sum in the last period. Of course, less extreme income paths may also generate overconsumption. For example, we calibrate that an agent who earned the median real wage between ages 20 and 65 and then retired until the median life expectancy of 78 would overconsume at age 20 for any preferences which generate a non-decreasing consumption plan.\footnote{We simplify this exercise by assuming the agent faces no risk and can freely borrow or save at a real interest rate of 5%.} As a consequence, at later ages the agent will underconsume from the perspective of the optimal lifetime consumption path. The wealth effect is therefore likely to dominate decisions in many real-world settings, and the exact value of the agent’s EIS is of second-order importance.

The magnitude of the agent’s misperception is a function of timespan (e.g. he makes no error regarding the present value of money from next period since compounding only occurs after spans greater than one), and so he will generally behave in a dynamically inconsistent manner. Conceptually, this can be distinguished from other varieties of dynamic inconsistency that are preference-based (Strotz, 1956; Loewenstein, 1987; Laibson, 1997; O’Donoghue and Rabin, 1999; Loewenstein et al., 2003; Kőszegi and Rabin, 2006), since this dynamic inconsistency is generated instead by perceptual errors regarding compounding interest. Of particular interest is the predictable pattern in which the dynamic inconsistency manifests.

**Proposition 4 (Dynamic Inconsistency)** If the agent has a negative level of savings at the end of period \(t < T - 1\),

\[
\sum_{s=0}^{t} (y_{ts} - c_{ts}) \Pi_{j=0}^{t} (1 + i_j) < 0
\]  

then the agent’s period-\(t\) plan of consumption will exceed the period-(\(t+1\)) plan in all periods i.e.

\[
\text{(7)}
\]
\( \hat{c}_{t,\tau} > \hat{c}_{t+1,\tau} \forall \tau > t \). If the inequality in expression (7) is reversed then planned consumption in \( t+1 \) will increase for all periods, \( \hat{c}_{t,\tau} < \hat{c}_{t+1,\tau} \forall \tau > t \), and if the balance equals zero planned consumption in \( t + 1 \) will be unaffected.

The proposition can be explained intuitively. Since the agent underestimates exponential growth, each period he will underestimate the change in his asset position. If his balance is positive then he receives an unexpected windfall and if the balance is negative he receives an unexpected loss. Since the agent’s perception of the period-\( T \) value of income received at some intermediate future period \( \tau \) depends only on interest rates between \( \tau \) and \( T \), the perception does not change over time. The only change in the perception of the budget is the growth of the current balance, and this surprise change to his current wealth causes him to shift his planned consumption vector in the same direction as the change.

This proposition implies that if a biased agent receives most of his income late in life, necessarily causing the agent to maintain a negative balance early in life to finance early consumption, then his projected consumption plans will always exceed his actual consumption. This can be particularly costly for the agent if he has the option to commit to lower bounds on his future consumption. This may manifest in the housing market, where housing is a consumption commitment. A homeowner will find it costly and difficult to decrease his housing consumption in the next period since this generally requires selling the home.

The agent will also underestimate the costs of debt, which can lead to a debt trap of sorts. Because the agent underestimates the speed at which a debt grows, he will underestimate both the size and the number of payments necessary to amortize a debt in a given amount of time. Let \( a(L, \vec{\hat{r}}; \alpha) \) be the agent’s perception of the periodic payment required to payoff a loan over \( T \) periods with principal \( L \), and with interest rate vector \( \vec{\hat{r}} \). The function \( a \) is defined as \( \hat{V}(< L, -a, \ldots, -a >, \vec{\hat{r}}, \alpha) \equiv 0 \) for an accuracy level \( \alpha \).

**Proposition 5 (Debt Repayment)** If \( \alpha < \alpha' \), then \( |a(L, \vec{\hat{r}}; \alpha)| < |a(L, \vec{\hat{r}}; \alpha')| \). Moreover, an agent with EGB (i.e. \( \alpha < 1 \)) will underestimate the number of periodic payments of a fixed size necessary to repay a given debt.
The proposition formally states the intuition introduced by Stango and Zinman (2009) that EGB can lead to excessive leverage. The agent believes that the periodic payment \( a(L, \vec{i}; \alpha) \) is the amount necessary to fully repay a debt of amount \( L \) in the specified number of periods. This perceived amount is strictly decreasing in the degree of bias. By underestimating compounding interest, the biased agent will underestimate the costs of holding debt leading to the various puzzles discussed earlier. While we do not focus on these predictions in this paper, we note that they are consistent with the findings of previous research. For example, Soll et al. (2011) find that US adults underestimate the number of payments needed to pay off a hypothetical credit card balance.

A corollary of Proposition 5 is that, given the repayment schedule for a loan, the agent will over-estimate the periodic interest rate. However, when the period is less than a year, the agent may still under-estimate the effective annual rate because when he estimates it from the periodic rate he does not account sufficiently for compounding. This second step of aggregation can explain the seemingly opposite finding in Stango and Zinman (2009) that 98% of people underestimate the interest rate when given a debt and a monthly repayment plan over one year.\(^{13}\) Our model predicts that an agent with EGB will under-estimate the equivalent annual rate when presented with the range of values they use. Moreover, when the term of a loan is shorter than a single period, the simple interest exceeds the periodic interest. Short-term loans with simple interest rates often imply exceedingly high periodic rates and as a consequence lenders will prefer to frame the short-term loans as simple interest.\(^{14}\)

This last result highlights the fact that a biased agent’s valuation of a particular debt or investment product can depend on how the product is framed. In particular, because the agent’s valuation is sensitive to the number of periods of compounding, descriptions of cash flows and interest vectors with equivalent present value can lead the agent to under-estimate the value when more periods are used (e.g. monthly vs. annual interest rates) – and vice-versa. Proposition 6 formalizes this intuition and shows that the distortion in the agent’s beliefs is bounded.

\(^{13}\)More formally, Stango and Zinman (2009) use two questions from the Survey of Consumer finances, which we can interpret in our framework as the agent solving for \( \hat{V}(< L, 0, \ldots, 0 >, < \vec{i}, \vec{i} >; \alpha) = \hat{V}(< 0, m, \ldots, m, >, < \vec{i}, \vec{i} >; \alpha) \) and then \( p(< i, \ldots, i >, 0; \alpha) = p(< 0, \ldots, 0, R >, 0; \alpha) \). While the agent will over-estimate \( i \) given \( L \) and \( m \), he may nonetheless underestimate \( R \).

\(^{14}\)We thank an anonymous referee for this observation.
Proposition 6 (Framing) Consider $\vec{y} = <y_0, 0>$ and $\vec{i} = <i_0>$ such that there is a single period of interest and $i_0 > 0$, $y_0 > 0$. Let $i_n = (1 + i_0)^{1/n} - 1$ denote the equivalent rate when interest is received at a frequency of $n > 1$ such that $\hat{V}_{0,n}(\vec{y}^{(n)}, \vec{i}^{(n)}; 1) = \hat{V}_{0,1}(\vec{y}, \vec{i}; 1)$, where $\vec{y}^{(n)} = <y_0, 0, ..., 0>$ and $\vec{i}^{(n)} = <i_n, ..., i_n>$. Then for all $\alpha < 1$: $y_0 < \hat{V}_{0,1}(\vec{y}, <i_{\text{min}}>; 1) < \hat{V}_{0,n}(\vec{y}^{(n)}, \vec{i}^{(n)}; \alpha) < \hat{V}_{0,1}(\vec{y}, \vec{i}; \alpha)$, where $i_{\text{min}} = \ln(1 + i_0)$.

Proposition 6 states that a biased agent will under-estimate the present value of a financial product when it is described in terms of shorter time periods (and hence more compounding), and vice versa. His beliefs are bounded strictly away from zero growth, however, even in the limit as interest compounds continuously. This proposition gives some insight into the types of frames often chosen in the market. Lenders are incentivized to choose as short a time-period as possible in order to minimize the perceived repayment – for example, the market leader in the UK payday-lending industry advertises its loans as “1 percent per day”.

For investments, firms are incentivized to maximize the perceived return for potential investors and thus choose relatively large frames. A striking example of this effect is the multi-year frame chosen by many structured deposits products, which offer for example a 5-year return of 30% rather than annual returns of 5.4%.

Finally, the model also predicts how a biased agent will combine interest rates over time. An agent with accurate perceptions knows that an interest vector is equivalent to the vector of its geometric mean. A biased agent, on the other hand, will tend to overestimate the constant-rate equivalent for a given interest vector.

Proposition 7 (Equivalent rates) Define $m(\vec{i}; \alpha)$ such that $p(<m(\vec{i}; \alpha), ..., m(\vec{i}; \alpha), 0) = p(\vec{i}, 0)$ for all $\vec{i} \in \mathbb{R}^T$. Then if $\alpha < \alpha'$, $m(\vec{i}; \alpha) \geq m(\vec{i}; \alpha')$. The inequality is strict if $\exists j, k$ s.t. $i_j \neq i_k$.

The proposition implies that a biased agent prefers a varying interest vector (i.e. not risky but simply changing over time) to an equivalent non-varying interest vector when saving. The premium that the agent places on the varying interest vector is increasing in his bias. We find strong evidence for this prediction in Section 4.2.3. Biased agents will also be insufficiently sensitive to periods with negative interest rates when evaluating assets. For example a fully biased agent prefers an asset

\footnote{Including fees, the representative APR is 5853\%.}
that has a 100% return followed by a -60% return, yielding a -10.6% net return, over one that does not grow.

3 Experiment 1: Lifecycle Consumption

The theoretical results in Section 2 predict that EGB can cause overconsumption and dynamic inconsistency in predictable ways. Experiment 1 is designed to test the predictions of the theory in a controlled simplified lifecycle-consumption environment. Our purpose is not to test subjects’ competency in this task *per se*. Maximizing an intertemporal utility function is challenging even when interest rates do not compound. Our purpose is instead to test comparative static predictions specific to our model: compared to a baseline behavior, do changes in the economic environment change behavior in the ways that our theory predicts? To use a concrete metaphor, our approach is not to test the null hypothesis that a sample of archers will hit the bullseye, but whether they systematically under-react to a crosswind.

The parameters of the environment are set such that the main predictions are threefold: the presence of compounding interest will cause overconsumption in early periods (Proposition 3), delaying the receipt of income increases overconsumption (Proposition 1), and when current assets are positive consumption plans will be lower than actual consumption, but when current assets are negative consumption plans will be higher than actual consumption (Proposition 4). Since the experiment is designed to understand how the environment affects consumption behavior, we will save our analysis of heterogeneity across individuals for our second experiment in Section 4.

Furthermore, even though the opportunities for learning are limited when it comes to retirement savings, they do exist. A self-reflective biased consumer may compare her past predictions with outcomes, and use this information to de-bias her future predictions. (Kaheman and Klein, 2009)

3.1 Design

Subjects were given a task to maximize an explicitly stated utility function subject to an intertemporal budget constraint. By using the induced-value approach that is standard in experimental research (Smith, 1976), we can generate very clear theoretical predictions on behavior that would
otherwise be impossible when people’s value for consumption is unobserved. In addition to the explicit instantaneous utility function, subjects were assigned an income vector, and interest rate vector, and had to choose how much income to consume each period. Savings and debts would accrue interest as a function of the interest rates.\footnote{Given the abstractness and unfamiliarity of such a problem to a typical subject in our sample, we framed the problem as a game to feed a “digital dog”. Subjects were allotted “bucks” (income) on various “days” (periods) which could be used to purchase “dog food” (consumption). The dog food generated “wags” of the tail (utils). The object to be maximized was the sum of “tail wags” (utils) over the course of all the days. By using a familiar context we expressed the basic features in an easily digestible manner and kept subjects engaged with the task. Moreover, the context abstracted away from rules-of-thumb sometimes applied to consumption (e.g. “save X%”) and focused subjects on the induced valuations.}

Since the purpose of the experiment was to test the role of EGB in a lifecycle-consumption environment, and not to examine whether people are capable of maximizing arbitrary utility functions \textit{per se}, we provided as many features as we could to help subjects understand the relationship between consumption and utility in the task, and hence their payment. We emphasized the concept of marginal utility and illustrated with several examples. We also provided on all screens the utility function itself, a graph of the utility function, and a calculator taking consumption as input and giving utility as output. Furthermore, subjects went through three training rounds with feedback about the optimal plan and how their choices fared relative to it. These training rounds each had a single positive interest rate, with the interest rate set to zero in all other periods. This familiarized subjects with the task and helped train them on the basics of utility maximization without directly training them on compounding.

The main task consisted of four “lifecycles”: combinations of an income vector, interest rate vector, and number of periods. A lifecycle of length $T$ required consumption choices in period 0 through $(T - 1)$, with any residual consumption allocated to period $T$ automatically. Subjects’ earnings were strictly increasing in the achieved utility, and all payments were made at the conclusion of the experiment. Thus regardless of subjects’ own utility and time preferences, a preference for money means their optimal actions were to attempt to optimize the consumption problem as stated in the experiment.

There were four lifecycles, all six periods long, $t \in \{0, \ldots, 5\}$. Lifecycle N (non-compounding) had $\vec{y}_N = < 100, 0, 0, 0, 0, 0 >$ and $\vec{i}_N = < 0, 0, 100\%, 0, 0 >$. There was no compounding interest.
in this lifecycle and so EGB is irrelevant. Behavior on this problem established a benchmark level of imprecision and bias that is unrelated to EGB. The three other lifecycles all have compounding interest. Lifecycle S (endowment at Start) had the same income vector as Lifecycle N and \( \vec{\bar{y}}_S = <75\%,...,75\%> \). Lifecycle E (endowment at End) had \( \vec{\bar{y}}_E = <0,0,0,0,0,500> \). Lifecycle M (endowment at Middle) again uses the same interest vector \( \vec{\bar{r}}_M = <75\%,...,75\%> \), but had an income vector \( \vec{\bar{y}}_M = <0,0,100,100,0,0> \). We use high interest rates for statistical power in decisions involving just a few periods. Lower interest rates would require a longer experiment or more subjects. If one interprets a period in the lifecycle to be a decade, however, then the real annual interest rate in Lifecycles S, M, and E corresponds to 5.75%, which is near real historical long-run stock market returns.\(^{17}\) The order in which the lifecycles were presented was random. Proposition 1 predicts that overconsumption will be greatest for Lifecycle E and least for Lifecycle S.

The experiment had two arms, and each subject was assigned to one arm only (see Figure 1). A subject in the static arm chose their consumption plan vector for all periods simultaneously.\(^{18}\) If consumption in a given period would exceed the true budget, then the computer bounded consumption such that the budget was fully expended and set consumption in all remaining periods to zero. Similarly, any remaining wealth in the last period was automatically used for consumption. Thus behavior in the static arm elicits the subject’s perceived optimal committed consumption plan.

In the static arm, each lifecycle was divided into 5 rounds. The first round is described above. In subsequent rounds, each lifecycle has its first period progressively truncated. For example, in the fourth round of lifecycle S, subjects made a 3-period plan given \( \vec{\bar{r}} = <75\%, 75\%> \) and \( \vec{\bar{y}} = <100,0,0> \). In every round of Lifecycle N and S, the consumer received a lump sum in their first period; in

\(^{17}\) We reiterate that our model is not neutral to relabeling period length. The results of this experiment are therefore likely different to what would have been obtained from running the much-longer 60-period experiment with 5.75% interest rates. Proposition 6 suggests that by aggregating rates into 10-year spans we expect a lesser effect of EGB relative to an annual framing thereby understating our results. Our goal, however, is to test the comparative static predictions of the model, not to generate data that is comparable to a typical American’s lifecycle per se, and so we prioritize statistical power by using very large interest rates.

\(^{18}\) Figure C.2 displays the user interface subjects see in the static arm. The table at the top of the screen indicates the income and interest vectors. The entries below allow the subject to input their committed consumption plan. Below that is a calculator that allows the subject to compute how consumption relates to utility, and a table and graph that further explicates this relationship.
Notes: Each round of length $T$ has $T + 1$ periods. Subjects in the static arm choose consumption vectors for a total of 20 rounds. Subjects in the dynamic arm choose consumption vectors with feedback on their current balance for a total of 4 rounds. The order of Lifecycles, and rounds within a lifecycle in the static arm, was randomized by subject.

In the dynamic arm, consumption was chosen sequentially each period. Subjects first chose the consumption in period 0. After submitting their answer they were informed of their current savings (or debt). They then chose consumption for the next period, and so on, receiving updated information about their asset position. As before, subjects could not exceed their budget, and any remaining money was automatically used for consumption in period $T$. Thus the dynamic arm elicits subjects perceived optimal consumption in the current period under an information regime that provides feedback about the budget each period. In contrast to the static arm, the dynamic arm does not elicit the subject’s initial consumption plans for future periods.

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The instantaneous utility function given to subjects was $u(x) = x^{\frac{1}{2}}$. We chose this as the utility function for two reasons. Given that this is a constant relative risk aversion utility function (CRRA) with an elasticity of intertemporal substitution of 2, Proposition 3 predicts overconsumption for any income vector. Furthermore, because CRRA utility functions are homothetic, the value of the income stream should have no effect on the proportion of the budget spent in each period for both an accurate agent nor a biased one. This allows for direct comparisons of behavior across different lifecycles without the confound of nonlinear wealth effects.\footnote{Eliminating wealth effects for a non-homothetic utility function is not an option because a subject facing lifecycles with the same lifetime wealth but different income vectors will perceive different lifetime wealth due to EGB.}

The homotheticity of the utility function allows us to compare consumption vectors chosen across lifecycles and for subjects in the static arm, across rounds within a lifecycle. Because the interest rate vector is the same in lifecycles S, M, and E, optimal consumption as a percentage of lifetime wealth will be the same. By comparing consumption vectors within a lifecycle we can see how consumption plans change as subjects progress through the lifecycle. We can also use a subject’s behavior in each round to construct a counterfactual consumption vector that represents what that subject would have done if his consumption plans were non-binding. To construct what we will call “simulated consumption”, we begin with $c_0$ from the full $T = 5$ round to construct consumption in $t = 0$. Because $c_0$ in the $T = 4$ round gives the fraction of wealth that would be consumed in $t = 1$ of the original $T = 5$ round, we can multiply the remaining lifetime wealth by this fraction to construct our simulated $c_1$. Then $c_0$ in the $T = 3$ round implies the fraction of wealth that would be consumed in $t = 2$, and so on.\footnote{We can only use this for lifecycles that have all the income received at the start; otherwise the comparison would require us to know the subject’s $\alpha$ to infer their perceived lifetime wealth. Thus we can only simulate for Lifecycles N and S.}

This laboratory experiment was conducted at the UCLA California Social Sciences Laboratory (CASSEL). Subjects were not provided with any tools, and calculators/cell phones were expressly forbidden. Subjects could earn up to $35 based on the quality of their responses, in addition to a $5 participation fee. After completing the experiment, one round was chosen at random by the computer. Subjects were paid based on how much additional utility their plan earned above the minimum achievable utility, as compared to how much additional utility would be achieved by the
optimal plan. Letting $u_a$ be the achieved utility, $u_o$ the optimal utility, and $u_m$ the minimum possible utility, a subject’s additional payment was given by $35 - 35 \cdot [1 - (u_a - u_m)/(u_o - u_m)]^{1/2}$. This payment rule provides strong incentives for accuracy, as returns are increasing as one approaches the optimal utility. The mean incentive payment was $18.46$: dynamic subjects averaged $17.68$, while static subjects (who could be paid for rounds with fewer periods) averaged $19.38$.

3.2 Results

The overall performance of subjects is shown in Figure 2, where we plot the median consumption paths achieved by static and dynamic subjects by lifecycle. We also plot the optimal consumption path and the consumption predicted for a fully-biased type who commits to a consumption plan chosen at $t = 0$. Panel (a) shows that subjects generally understood the task in the non-compounding Lifecycle N, although there was a tendency to smooth somewhat too much. We therefore use this lifecycle to control for subjects’ performance in the absence of exponential growth, which raises the hurdle for our tests of EGB. Panel (b) shows that subjects overconsumed relative to the optimum in period-0, and that both dynamic and static subjects revised consumption up relative to this initial plan. Comparing panel (c) and (d) to panel (b) shows that delays in income result in dramatic increases in period-0 overconsumption. Subjects again smoothed planned consumption somewhat too much in panel (d), leading them to reach the budget constraint even more quickly than a fully-biased utility optimizer — but the qualitative prediction of resource exhaustion for agents with EGB remains a much better match to the data than is the optimal consumption path.

In the remainder of this section, we will address these features in more detail.

We begin by testing Proposition 3, that exponential-growth bias leads to overconsumption in early periods relative to the optimal spending path. We focus on consumption in the initial period both because of the direct relation to the Proposition, and to eliminate the effect of feedback in later periods for subjects in the dynamic arm. Additionally, a comparison of actual consumption to the ex ante optimal consumption in later periods is difficult to interpret due to the confounding effect of prior periods’ consumption. For example, if a subject consumes the whole budget in $t = 0$, consumption in all subsequent periods will be constrained to 0, but one should not then
Figure 2: Feasible Consumption Paths

(a) Lifecycle N: No compounding.

(b) Lifecycle S: Compounding, income at start.

(c) Lifecycle M: Compounding, income at middle.

(d) Lifecycle E: Compounding, income at end.

Notes: Median consumption paths achieved (i.e. subjects’ plans restricted to feasible consumption), by lifecycle. Static refers to the consumption path implemented by the \( T = 5 \) plan of subjects in the Static arm; Dynamic refers to the consumption path implemented by subjects in the Dynamic arm; Optimal is the consumption path predicted for an unbiased utility optimizer; Fully Biased is the consumption path predicted for a fully exponential-growth biased utility optimizer. In panels (a) and (b), simulated refers to our simulation of static subjects’ re-optimization based on their choices in all rounds.
interpret this as evidence for underconsumption when examining later periods \( t \geq 1 \). That said, the progressive truncation of the lifecycle in the static arm still allows us to look at behavior with different horizons.

Figure 3 plots the distribution of the natural logarithm of the ratio of subjects’ period-0 consumption to the optimal period-0 consumption for that lifecycle: \( \ln(c_0/c_0^*) \). This provides a simple measure of over-consumption: subjects over-consume relative to the optimum when this variable is positive, and under-consume when it is negative. For simplicity, we pool static and dynamic subjects’ answers, restricting to the \( T = 5 \) rounds for static subjects for comparability. In panel (a) of Figure 3, we plot the distribution of the log-ratio for Lifecycle N, which has only a single nonzero interest rate and therefore EGB does not predict overconsumption. While the median value is higher than zero, the overall distribution indicates only a minor tendency to over-consume in period 0. Any overconsumption in panel (a), however, is dwarfed by the overconsumption shown in panel (b), where we present the distribution of the log-ratio for Lifecycles S, M, and E, which have more than one period of non-zero interest. The bias towards overconsumption here is apparent, and both the mean and median are significantly more positive than without compounding.

We unpack the sources of overconsumption in Table 1. We use the same dependent variable,
\[ \ln(c_0/c_0^*) \] as in Figure 3, and use OLS specifications in all columns. Using an alternative transformation \[ \ln((c_0 + 1)/(c_0^* + 1)) \], which does not drop zeroes, does not qualitatively change any results.

Column 1 of Table 1 confirms that the presence of multiple non-zero interest rates significantly increases the initially mild overconsumption of subjects. The variable “Compounding” is a dummy variable that takes on the value one for the Lifecycles with compounding interest: S, M, and E. Subjects consume approximately 350\% more in period 0 than they did in the non-compounding control, Lifecycle N. In column 2, we see that this overconsumption is driven by the longer rounds. The variable “Extra Periods” records the number of periods by which the round is longer than 2 periods. With just 2 periods (T=1) there is of course no possibility for EGB to distort decision-making, and we find that the “compounding” indicator is indeed not significantly different from zero. As the number of periods grows, there is a small but significantly positive increase in the level of overconsumption. The quantitatively more important effect, however, is the interaction of “Extra Periods” with the compounding dummy, which indicates that adding an additional decision-making period in a lifecycle with compounding interest rates of 75\% leads to roughly a 50\% increase in the ratio of period-0 consumption to its optimal value. This is exactly the prediction of the theory, as subjects’ perceptions of the cost of period-t consumption are increasingly distorted for larger t.

In columns 3 and 4, we restrict attention to the longest rounds (T=5), yielding one observation per lifecycle for both static and dynamic subjects. In column 4, we investigate whether the two groups behave differently in their choices of \( c_0 \). Because dynamic subjects have not received any additional feedback until after they have chosen \( c_0 \), we would not predict any systematic difference. Neither the indicator for Dynamic nor the interaction of that indicator with the dummy for multiple non-zero interest rates is significantly different from zero.

Finally, column 5 of Table 1 asks whether subjects learn to correct their bias over the course of the experiment. We restrict the sample to those lifecycles with multiple non-zero interest rates, and

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\(^{22}\) LAD estimates for the effects on the median, as well as regressions of levels rather than logs, are available for all tables in this section, and do not substantively alter any results.

\(^{23}\) There are 38 subjects in the static arm each generating a consumption vector for each of 5 rounds for each of 4 lifecycles producing a total of 760 observations, and 44 subjects in the dynamic arm each generating a consumption vector for each of 4 lifecycles generating a total 176 observations. We drop the 53 observations for which \( c_0 = 0 \) for a total of 883 observations. [This was in the text, and the text sentence was the footnote]
Table 1: Effect of Compounding on Overconsumption

<table>
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Notes: Dependent variable is $\ln(c_0/c^*_0)$; all columns are OLS regressions. Extra periods is the number periods by which $T > 1$. Question refers to the order in which the subject answered the question; subjects in the static arm answered 20 questions excluding the training, while subjects in the dynamic arm answered 4 excluding training. Standard errors clustered by subject. *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$
include all rounds for static subjects. Because the lifecycles were presented in a random order, as were the multiple rounds within a lifecycle for static subjects, we can include a variable “Question” which records the order in which a subject actually answered that particular round. We would not expect any learning among static subjects, since they received no feedback, but it is possible that dynamic subjects would learn about their EGB and revise their behavior accordingly. However, neither the Question variable nor any of its interaction terms are significantly different from zero, suggesting that subjects did not learn to perform better over the course of the experiment. Given the limited feedback and moderate stakes, however, it remains plausible that biased consumers may nevertheless learn to mitigate the effects of EGB when maximizing their own utility in their life.

We next turn to our second hypothesis for this experiment. From Proposition 1, we have that delaying the timing of income will exacerbate overconsumption of EGB individuals. The simplest test of this prediction, then, is to compare the degree of overconsumption in period 0 from Lifecycle S, where all income is received in period 0, to that from Lifecycle E, where all income is received in period T. Proposition 1 predicts that overconsumption will be greater when all income is received at the end of the lifecycle.

In Table 2, we regress our measure of overconsumption, $\ln(c_0/c_0^*)$, on an indicator for whether income is delayed, using rounds of all lengths from Lifecycles S and E and pooling dynamic and static subjects together.\footnote{There are 38 subjects in the static arm each generating a consumption vector for each of 4 rounds (the shortest 5th round does not have compounding and is excluded) for each of 2 lifecycles (S and E) producing a total of 304 observations. A truncated 3-period round of Lifecycle M also has all income at the start so these are also included increasing the number of observations by an additional 38. In the dynamic arm there are 44 subjects each generating a consumption vector for each of 2 lifecycles generating a total 88 observations. Of the 430 observations we drop the 18 observations for which $c_0 = 0$.} In column 1, the estimate of 1.255 on the delayed income dummy variable is highly significant and confirms the prediction of the theory. Delaying income causes an approximate increase in consumption by about 250%. Column 2 interacts the delayed income dummy with the number of periods in the round. The coefficient of 0.445 on the Extra Periods variable reiterates the finding from Table 1 that overconsumption is increasing in the number of periods. The interaction of the delayed income dummy with the number of periods is also positive at 0.250 and highly significant. This prediction follows as a corollary of Proposition 1.

In columns 3 and 4 of Table 2, we restrict the sample to those rounds with $T=5$, and add
Table 2: Effect of Delayed Income on Overconsumption

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<td>All</td>
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<td>T=5</td>
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<tr>
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<td>1.316***</td>
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<tr>
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<td></td>
<td>(0.059)</td>
<td></td>
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<tr>
<td>Extra Periods</td>
<td></td>
<td>0.445***</td>
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<td>(0.036)</td>
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<tr>
<td>All At Middle</td>
<td></td>
<td></td>
<td>0.288**</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.113)</td>
<td></td>
</tr>
<tr>
<td>Dynamic</td>
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<td>-0.212</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td>(0.166)</td>
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<td>At End X Dynamic</td>
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<td></td>
<td></td>
<td>(0.250)</td>
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<td>All at Middle</td>
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<td>0.125</td>
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<td>(0.138)</td>
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<tr>
<td>At Middle X Dynamic</td>
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<td>0.306</td>
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<td></td>
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<td></td>
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<td>Constant</td>
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<td>0.207*</td>
<td>2.009***</td>
<td>2.122***</td>
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<tr>
<td></td>
<td>(0.071)</td>
<td>(0.116)</td>
<td>(0.083)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>N</td>
<td>412</td>
<td>412</td>
<td>221</td>
<td>221</td>
</tr>
<tr>
<td>Subjects</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is ln(c₀/cₜ₀); all columns are OLS regressions. For columns 1-2 sample comprises those lifecycles where the entire endowment is received either in period 0 (All At Start) or period T (All At End). For columns 3-4 all lifecycles of length T = 5 are used. Standard errors clustered by subject. *p < 0.1; **p < 0.05; ***p < 0.01
Lifecycle M where all the income is received in the middle periods. There are 82 subjects with 3 lifecycles each generating 246 observations, and we drop the 25 observations for which \( c_0 = 0 \). This ensures that both static and dynamic subjects have two observations each, and confirms that the effect is largest in the longest rounds. That the coefficient on Lifecycle M should be positive and less than the coefficient on Lifecycle E is an even stronger test of Proposition 1, and is confirmed (we reject equality of the coefficients at \( p < 0.01 \)). This follows because the perception of the value of income received three periods in the future is less distorted by EGB than that of income received five periods in the future. Finally, the last column confirms that there is no systematic difference between subjects in the two treatments: neither the dynamic dummy nor its interaction with the delayed income dummy enters significantly.

Finally, we address our prediction of dynamic inconsistency from Proposition 4. Once again it is easiest to test this hypothesis by restricting attention to Lifecycles S and E. Because Lifecycle S has all income received in period \( t=0 \), subjects will always carry a weakly positive balance. Proposition 3 suggests that dynamic subjects will be surprised at how quickly their balance grows, and will revise their consumption upwards relative to their initial plan in later periods. Conversely, subjects in Lifecycle E must borrow against their income from the final period to finance early consumption, and should revise their consumption downwards in later periods. We can test these predictions by using static subjects’ plans from the \( T=5 \) rounds as proxies for dynamic subjects’ initial plans. That is, we observe dynamic subjects’ initial choice only for \( c_0 \), but not their planned choices for \( c_1 \) through \( c_5 \) that led them to believe that this choice of \( c_0 \) was optimal. Static subjects, however, were asked for a full consumption plan without the opportunity for any revisions out – and did so for each of 5 rounds per lifecycle. We can therefore compare dynamic subjects’ actual consumption to the feedback-free consumption plans of static subjects in order to measure the effect of feedback about the remaining budget.

We use OLS regressions to perform these comparisons in Table 3. The dependent variable is now the natural logarithm of period-\( t \) consumption normalized by lifecycle wealth, to account for the higher wealth in Lifecycle S. \(^{25}\) The first column pools Lifecycles S and E, and finds no significant

\(^{25}\) Each of the 82 subjects produces an observation per period for each of 5 periods for each of 2 lifecycles for a total of 820 observations. Observations with \( c_0 = 0 \) are dropped for a total of 763 observations.
differences between the static and dynamic subjects’ choice for $c_0$, although the standard errors are quite large. The coefficient of 0.091 on the period variable indicates that subjects correctly plan for an upward-sloping consumption profile, although the trajectory is flatter than would be optimal. Columns 2 and 3, which separate out Lifecycles S and E respectively, confirm the predictions of Proposition 3. The Dynamic indicator is again insignificant in column 2, as dynamic and static subjects do not differ in their plans for $c_0$. In later periods, however, dynamic subjects are evidently surprised to learn how much their savings have grown. The coefficient on the Dynamic X Period interaction is large and positive, indicating that dynamic subjects’ actual consumption choices in later periods are substantially higher than static subjects’ consumption plans. In each subsequent period, dynamic consumption increases relative to the static consumption of the same period by about 29% (e.g. after 2 periods the dynamic consumption has grown $e^{0.257 \times 2} \approx 67\%$ relative to the static consumption level). The opposite pattern emerges in column 3, where Lifecycle E dynamic subjects are surprised by how little they have left to spend. The Dynamic X Period interaction is again large and significant, but now negative – indicating that dynamic subjects’ actual consumption choices are revised substantially downwards from static subjects’ plans at $t=0$. Here, dynamic consumption decreases relative to the static consumption of the same period by about 21%.

Strikingly, this revision by dynamic subjects is no more than we would have expected from static subjects’ choices in shorter rounds. While we can only simulate how static subjects would have re-optimized in Lifecycle S, the simulated static consumption without commitment is indistinguishable from the dynamic consumption. This was seen clearly in Figure 2, panel (b).\textsuperscript{26} This implies that feedback on one’s balance has no impact on consumption beyond the prediction of Proposition 4. Whereas the previous results from Table 1 show that there is no learning across the experiment (one cannot reject the null that the coefficient on the “Question” order variable is zero), this result shows that there is also no learning within a lifecycle.

Finally, we note that to earn a reasonable payment, subjects simply had to save enough so that

\textsuperscript{26}Using the simulated static consumption without commitment and dynamic consumption data, we can regress consumption on a set of time dummies interacted with a “dynamic” dummy. We cannot reject that the time dummies interacted with dynamic are jointly different from zero ($F(5, 81) = 0.41$, $p=0.840$).
Table 3: Dynamic vs. Static Plans

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Inc At Start</td>
<td>Inc At End</td>
</tr>
<tr>
<td>Dynamic</td>
<td>-0.213</td>
<td>-0.211</td>
<td>-0.138</td>
</tr>
<tr>
<td></td>
<td>(0.162)</td>
<td>(0.155)</td>
<td>(0.250)</td>
</tr>
<tr>
<td>Period</td>
<td>0.091***</td>
<td>0.109**</td>
<td>0.085**</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.048)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Dynamic X Period</td>
<td>0.030</td>
<td>0.257***</td>
<td>-0.242***</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.076)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.877***</td>
<td>-1.540***</td>
<td>-0.245</td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.117)</td>
<td>(0.181)</td>
</tr>
<tr>
<td>N</td>
<td>763</td>
<td>398</td>
<td>365</td>
</tr>
<tr>
<td>Subjects</td>
<td>82</td>
<td>81</td>
<td>80</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is $\ln(c_t/W_0)$, i.e. consumption normalized by actual starting wealth. The sample comprises the compounding T=5 lifecycles with all income received either in period 0 or in period 5. One subject planned consumption $c_t = 0$ for periods 1-4 when income at start; two (different) subject planned consumption $c_t = 0$ for periods 1-4 when income at end. Since the $\ln(0)$ is undefined these subjects are dropped. Standard errors clustered by subject. ∗$p<0.1$; ∗∗$p<0.05$; ∗∗∗$p<0.01$

they would have nonzero consumption every period. It was on this front that both dynamic and static subjects failed. Largely due to their $c_0$ choice, 36.0% of static subjects and 34.8% of dynamic subjects’ $T = 5$ plans from Lifecycles S, M, and E led them to exhaust all their resources early (i.e. $c_5 = 0$). This rose to 73.7% and 68.2% in Lifecycle E, of which 63.2% and 59.1% respectively occurred by the end of the second period. While dynamic subjects realized their debts were growing faster than they anticipated, in effect it was too late for them to get back on track.

4 Experiment 2: EGB in the U.S. Population

In our second experiment, we examine whether our parameterized model of EGB is a useful measurement that has meaning outside the laboratory. We shift from a lifecycle consumption paradigm to a direct perception elicitation in order to directly estimate the EGB parameter $\alpha$ with the fewest potential confounds. The experiment has several purposes. First, we estimate the distribution of EGB among a representative subsample of the U.S. population, making decisions in their normal work environment. Second, the experiment tests the external validity and relevance of EGB in economic decision making by correlating it with total savings. We interpret this not as a further direct
test of the theory (Propositions 2 and 3) due to substantial unobserved heterogeneity, but rather as an indication that our measure of EGB can be usefully applied to explain some of the variation in financial decision making. Third, the experiment tests whether the bias is robust to a simple graphical intervention. Fourth, the experiment tests specific features of our model in contrast to the Wagenaar-Sagaria model, namely: (1) people correctly estimate a single period of interest, (2) people underestimate the value when there is negative interest, (3) people combine interest rates in a way that biases them toward the arithmetic mean (Proposition 7), and (4) people do not decompose financial problems into a series of single periods.

4.1 Design/Method

4.1.1 Design

Subjects faced a series of questions describing two assets and were asked to indicate the initial value for one asset which would equate the assets’ final values after an indicated length of time. For example, the first question is a choice between “Asset A that has an initial value of $100 and grows at an interest rate of 10% each period” and “Asset B that has an initial value of $X and does not grow.” Subjects were asked for the value of X which would make the two assets equal value after 20 periods. Appendix Table C.2 displays the full list of questions presented to subjects.

Questions 1–10 are our primary focus in the analysis, and the order of presentation was randomized first at the domain level (corresponding to the first 3 question categories listed in Appendix Table C.2) and then within domain at the question level. The “exponential” domain comprised four questions similar to the example above. The “fluctuating-interest” domain comprised three questions of the form: “Asset A has an initial value of $P and grows at an interest rate of i% in odd periods (starting with the first) and at j% in even periods; Asset B has an initial value of $X and does not grow; What value of X will cause the two assets to be of equal value after T periods?”.

The “catch-up savings” domain comprised three questions which varied the maturity of the assets, of the form: “Asset A has an initial value of $P and grows at an interest rate of i% each period; Asset B has an initial value of $X and grows at an interest rate of i% each period; What value of X will cause the two assets to be of equal value after Asset A grows for T periods and Asset B
grows for $S$ periods?"

Subjects received a payment based on their accuracy on each question. The payment rule was piecewise-linear in the percentage error: each answer within 10% of the truth would receive $0.80; each answer within 25% would receive $0.60; each answer within 50% would receive $0.20; and each answer less than 50% of or more than 150% than the truth would not receive a payment. In addition to the incentive payments, subjects received $5.00 for completing the entire experiment. Subjects had a week to do the experiment at their leisure. All payments were made through Knowledge Networks’ internal payment mechanism, which subjects were already experienced with, and were usually paid within a week of completion.

The experimental instrument intentionally did not mention the use of tools for answering the questions. Subjects could potentially use whatever tools that they had access to: from nothing to advice from friends or financial calculators. This design neither discouraged subjects’ natural tendency to use tools nor did it prime subjects to use them. Although the incentives are not nearly as large as they would be for making actual financial decisions in the marketplace, a subject that exploited tools could earn substantially more for their time.

After completing the primary experiment, subjects were randomly assigned into a control (N=384) and a treated (N=185) group to test the effect of a simple information presentation or “nudge” on a second set of questions. The intervention, shown in Appendix Figure C.4, shows the growth of $100 at one or more relevant interest rates, and allowed the subject to specify the time horizon plotted. Treated subjects were shown this intervention beneath each question. Subjects answered an additional 16 questions, 10 from the original three diagnostic domains and 3 each from the domains of periodic savings – asking the final value of a series of regular contributions – and portfolio – asking the equivalent principal for a portfolio of assets at different rates. These latter two domains are qualitatively more complicated to solve.27 We exclude these additional questions from all analysis until Sections 4.2.3, focusing just on the 10 pre-intervention questions where all subjects faced identical circumstances.

27We do not formalize a metric of “difficulty”, but note that, for example, Catch-up Savings questions require a subject to sum $T$ separate Exponential questions.
4.1.2 Sample

Experiment 2 consists of an incentivized online experiment conducted on a nationally-representative sample. Participants were recruited through Knowledge Networks, which maintains a recruited panel of U.S. households. A random sample of subjects from the Knowledge Networks panel were invited to participate in our study. Subjects logged into the Knowledge Networks portal, and were automatically transferred to an external website where our study was hosted.

Table 4 shows summary statistics for our sample. Column 1 shows the characteristics of all 990 KN panelists who were invited to participate, while Column 2 comprises the 569 subjects who chose to participate. Men were significantly more likely to complete the study (63% vs. 52% for women, $p<0.01$), so that 46% of the final sample were women. The average age of those opting to complete the study was also somewhat lower than those opting out, although this result was largely driven by a very high completion rate amongst 18–21 (i.e. college-aged) panelists. Race and education characteristics did not predict study completion, with 28% of subjects having only a high school degree, 29% some college or an associate’s degree, and 37% having a bachelor’s degree or more.

For some of the analysis, we merge our experimental dataset with an external dataset containing subjects’ financial characteristics. Participants in the Knowledge Networks panel are regularly asked about their income and assets, and we will use this information to investigate the effect of exponential-growth bias on savings. These data are only available for a subset of subjects (the others either being ineligible or refusing to answer), and we present them in the fourth column of Table 4. Unsurprisingly, this subsample tends to be older and better educated than those for whom financial data are unavailable: the mean age is 50.02, and 53% have at least a bachelor’s degree. The higher education attainment rate is also reflected in the high average household income of $90,257 among this group. This group also had significant investible assets — a mean of $241,055 — suggesting that they could overstate the degree of financial sophistication relative to a poorer, less well-educated population. However, we find that the sample with complete financial data are

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28Participant households are selected randomly by Knowledge Networks based on their address, and are provided with a laptop and free internet access if necessary. Full details on the KnowledgePanel sampling methodology are available at http://www.knowledgenetworks.com/knpanel/KNPanel-Design-Summary.html
in fact slightly more biased than the sample with missing financials – the partial correlation of our \( \alpha \) measure with the absence of financial data is 0.024 (\( p<0.01 \)).

Table 4: Summary Statistics — Experiment 2

<table>
<thead>
<tr>
<th>Demographics</th>
<th>Invited Sample</th>
<th>Study Completers</th>
<th>Completers w/o Assets Data</th>
<th>Completers with Assets Data</th>
<th>US Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>0.51</td>
<td>0.46</td>
<td>0.47</td>
<td>0.46</td>
<td>0.52</td>
</tr>
<tr>
<td>Age</td>
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<td>44.73</td>
<td>39.00**</td>
<td>50.02***</td>
<td>45.21</td>
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<tr>
<td></td>
<td>(17.30)</td>
<td>(17.17)</td>
<td>(17.41)</td>
<td>(15.15)</td>
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<tr>
<td>Education</td>
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<td></td>
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<tr>
<td>Completed High School</td>
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<td>0.28</td>
<td>0.37</td>
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<td>Some College</td>
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<td>0.27</td>
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<td>Bachelor's Degree+</td>
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<td>0.37</td>
<td>0.20***</td>
<td>0.53***</td>
<td>0.28</td>
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<td>Race/Ethnicity</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Black, Non-Hispanic</td>
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<td>0.07*</td>
<td>0.08</td>
<td>0.06</td>
<td>0.14</td>
</tr>
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<td>0.05**</td>
<td>0.04</td>
<td>0.07</td>
<td>0.08</td>
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<tr>
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<td>0.13**</td>
<td>0.17</td>
<td>0.10*</td>
<td>0.18</td>
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<tr>
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<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
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<td>0.70</td>
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<tr>
<td>Has used payday loan</td>
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<td>0.05</td>
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<tr>
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<td>132822</td>
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</tr>
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<td></td>
<td></td>
<td></td>
<td>(375913.20)</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>68763.74***</td>
<td>90257.60***</td>
<td>69759</td>
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</tr>
<tr>
<td></td>
<td>(50224.95)</td>
<td>(49887.68)</td>
<td>(48371.71)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>990</td>
<td>569</td>
<td>273</td>
<td>296</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Initial sample comprises all subjects invited to participate in the study. Study completers answered or skipped all questions. Assets data merged from external dataset provided by Knowledge Networks. US Population is based on authors calculation from 2014 Current Population Survey (and SCF for asset data). Table entries are unweighted means; standard deviations for non-binary variables given in parentheses. Stars denote significant difference from US population average, using study weights; *\( p < 0.1 \); **\( p < 0.05 \); ***\( p < 0.01 \)

4.2 Results

We begin this section by showing evidence that subjects in Experiment 2 were systematically affected by exponential-growth bias despite the availability of financial tools outside the laboratory.
We then estimate an individual-level bias parameter based on the model presented in Section 2, and investigate its distribution and correlation with household finances.

4.2.1 Bias

For each subject $i$ and exponential-growth question $j$, we first calculate the natural logarithm of the ratio of the given answer to the correct answer. Let a subject’s responses on question $j \in \{1, \ldots, J\} = \mathcal{J}$ be denoted by $r_{ij}$, and the correct response be given by $c_j$. We calculate $e_{ij} = \ln(r_{ij}/c_j)$. This provides a consistent measure across questions that may have answers that differ by several orders of magnitude. Were a subjects to answer exactly correctly, this statistic would be exactly zero. As subjects were not prohibited from using calculators, spreadsheets, and online tools to help them answer the questions, such an outcome would not strain credulity. If subjects’ answers are unbiased but noisy such that errors on an absolute or percentage basis are symmetrically distributed around zero, then the median of $\ln(r_{ij}/c_j)$ should also be zero. Moreover, if subjects’ answers are a power of the correct answer, $r_{ij} = c_j^{1+\varepsilon_{ij}}$ where $\varepsilon \sim \mathcal{N}(0, \sigma^*)$, then the log-ratio should be normally distributed about zero.

We also calculate subject-level averages of the above log-ratio:

$$\bar{e}_i = \frac{1}{10} \sum_{j=1}^{10} \ln(\text{answer}_{ij}/\text{correct}_j)$$

If subjects are making unbiased errors, then the above results for the means and medians hold. Moreover, if $r_{ij}/c_j$ is i.i.d. lognormal, then the averaging should cause the distributions to collapse towards zero.

Instead, we find a systematic bias in the error, the sign of which depends on whether exponential-growth bias predicts that subjects should over- or under-predict on that question. Figure 4 plots the distribution of log errors at the question×subject and subject level. As expected, the modal question×subject error is zero – the likeliest interpretation is that a large mass of subjects are able to use calculators to get the answer exactly correct. However, where under-estimation is predicted, both the median (-0.349) and mean (-0.554) of the question×subject error distribution are significantly negative (p<0.01). At the subject level, both the median (-0.507) and mean
(-0.602) are more negative than before. The pattern is reversed where exponential-growth bias predicts over-estimation: the question×subject distribution is shifted sharply to the right and the mean error (0.209) is now significantly positive (p<0.01), although the median error in this case is zero.\footnote{These results imply that pooling all questions by reversing the scale on the underestimation-prone items would also produce a significant result. At the subject level, both the mean (0.400) and the median (0.405) are significantly positive.}

Figure 4: Subject Mistakes

4.2.2 Estimating Alpha

Let $\tilde{a}(\alpha) : \mathbb{R} \rightarrow \mathbb{R}^{\lvert J \rvert}$ be a function that generates the answers consistent with a given level of $\alpha$ on a set of questions $J$. Thus $\tilde{a}(1)$ is a vector containing the $\lvert J \rvert$ correct answers. For every subject, we calculate the value of $\alpha$ which minimizes the mean squared error of the model against their actual answers, with each question normalized by the correct answer. This normalization avoids having those questions which contain large values for the solution arbitrarily dominate the
estimation procedure. That is, we estimate:

$$\hat{\alpha}_i = \arg \min_{\alpha} \frac{1}{|\mathcal{J}|} \sum_{j \in \mathcal{J}} \left( \frac{r_{ij} - a_{j}(\alpha)}{a_{j}(1)} \right)^2$$

The estimator described by (8) is not constrained to values lying within the unit interval. Values of $\alpha$ greater than one are simply interpreted as an individual who overestimates the rapidity of exponential growth. We perform an unconstrained numerical optimization to estimate an $\hat{\alpha}_i$ for each subject.

Figure 5 plots the cumulative distribution of our estimates of $\alpha$, using our full sample of completers. We characterize 85% of the population with an $\alpha$ between $[0, 1]$. The median $\alpha$ is 0.53, and the mean is 0.60. Moreover, we have a large number of people who are completely, or nearly completely, fully biased: 33% of subjects (184/561) have an alpha of “exactly” zero (i.e. within $[-0.001, 0.001]$). In contrast, only 4% (23/561) are completely correct (even using a more generous interval of $[0.99, 1.01]$). Based on our bootstrapping procedure, we can reject that the 80th percentile has $\alpha = 1$ at 95% confidence. Similarly, we cannot reject that the 37th percentile has $\alpha = 0$.

Although EGB appears to be pervasive we qualify this with two major caveats. The stakes in our experiment are much lower than in many financial decisions. Second, when horizons are short and interest rates are in the single digits, people in the upper quartile of $\alpha$ will not make very large errors. For instance with a 20 period horizon and 5% periodic interest rate, a 0-Eddie will underestimate the value of an asset by 25% but a 0.9-Eddie will only underestimate by 5%. On the other hand, even a 0.9-Eddie will make substantial errors in some situations, for instance the 0.9-Eddie will underestimate the annual equivalent of a loan that charges 1% a day by 30 percent.

Question 3 gives us a test of basic interest-rate numeracy. The question asks for the value of an asset after it grows for only one period ($P_0 = 100, i = 4\%$), and a correct answer would result from any degree of bias. A wrong answer can thus be interpreted as innumeracy. About 74% of our subjects answered this question correctly and 26% did not. Dropping subjects who fail to answer this question correctly does not substantively change any of the remaining analysis. Moreover, mistakes on this question are uncorrelated with our measure of $\alpha$ ($r = 0.04$, $p=0.29$)
which provides reassurance that we are estimating a systematic bias rather than understanding of the task. Almenberg and Gerdes (2012) find that EGB and financial literacy are negatively correlated in contrast to what we find here. These results are not inconsistent, however, given that their measure of financial literacy is quite different from our Question 3.

Our measure of $\alpha$ is uncorrelated with education, age, race, and sex. Unsurprisingly, the 6% of subjects who reported an online calculator perform substantially better than the rest of the population. The mean $\alpha$ in this group is 0.84 (0.32 higher than those who do not use financial calculators) and with a median $\alpha$ of 0.96 (relative to a median amongst those who do not use financial calculators of 0.56). There may be both a causal and self-selection effect in this population, which our research design does not distinguish. The use of calculators may also help to explain why our graphical treatment had no effect, as we explain later.

With estimates of individuals’ $\alpha$ we can explore the relationship between EGB and long-run financial outcomes. Proposition 3 states that biased agents will systematically overconsume in early periods relative to the optimal consumption path when the elasticity of intertemporal substitution is
greater than unity. This posits exponential-growth bias as a partial explanation for the high degree of variation in retirement savings within income and education categories found by Bernheim et al. (2001) and supported by the correlation found in Stango and Zinman (2009). While we do not have exogenous variation in $\alpha$ we can correlate $\alpha$ with total accumulated assets.

We are able to match financial records from an external survey to 296 of our 569 experimental subjects. As the unmatched cases correspond to refusals or ineligible cases (often college students), this leaves us with a slightly older and better-educated subsample. We then estimate the relationship between our estimate of $\alpha$ and investible assets. We perform regressions of the form:

$$assets_i = \theta_1 + \theta_2 \cdot \hat{\alpha}_i + \theta_3 \cdot income_i + \theta_4 \cdot age_i + \theta_5 \cdot male_i + \theta_6 \cdot hhsize_i + \theta_7 educ_i + \varepsilon_i$$ (9)

where assets and income are measured either as the level or natural logarithm.

The results of this regression are shown in Table 5. Columns 1 and 2 present the results where $\ln(assets_i)$ is the dependent variable, and columns 3 and 4 present the results where $assets_i$ is the dependent variable. In columns 1 and 3 subjects are uniformly weighted whereas in columns 2 and 4 subjects are weighted by study-specific post-stratification survey weights that re-balance the sample to national representativeness. All models have state fixed effects. Unsurprisingly, older people have accumulated more assets, and a 1% increase in income tends to be associated with slightly more than 1% higher level of savings. Although education is positively associated with higher total savings, it is only marginally significant.

Our coefficient of interest, Alpha, is positive and substantial throughout all the models. The estimated coefficient in logs is 0.438 and 0.640 in the two models and both are highly significant. The magnitude of the effect is large: all else equal, an un-biased agent will accumulate from 55% (column 1) to 90% (column 2) more assets than a fully biased agent. We caution that this association is not causally identified, but note that there are several plausible channels including the over-consumption result of Proposition 3 and the mis-use of credit of Proposition 5, as well as possible reverse-causation. Translated into levels in columns 3 and 4, this effect is equivalent to $87,877.807$ and $93,499.748$ respectively, or approximately one third of the median household’s non-annuitized wealth at retirement (Poterba et al., 2011). We thus find that EGB can help to
predict the large differences in otherwise equivalent households’ wealth.

Table 5: Alpha and Savings

<table>
<thead>
<tr>
<th></th>
<th>Logs (1)</th>
<th>(2)</th>
<th>Levels (3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>0.438**</td>
<td>0.640***</td>
<td>87,877,807**</td>
<td>93,499,748***</td>
</tr>
<tr>
<td></td>
<td>(0.178)</td>
<td>(0.215)</td>
<td>(39,269,207)</td>
<td>(31,057,441)</td>
</tr>
<tr>
<td>Ln(Income)</td>
<td>1.114***</td>
<td>1.201***</td>
<td>21,507,573*</td>
<td>16,636,850</td>
</tr>
<tr>
<td></td>
<td>(0.159)</td>
<td>(0.149)</td>
<td>(12,682,178)</td>
<td>(13,485,775)</td>
</tr>
<tr>
<td>Education</td>
<td>0.078</td>
<td>0.023</td>
<td>10,639,556***</td>
<td>10,720,404***</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.058)</td>
<td>(1,385,969)</td>
<td>(1,407,761)</td>
</tr>
<tr>
<td>Age</td>
<td>0.059***</td>
<td>0.058***</td>
<td>-24,140.997</td>
<td>-28,202.743**</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(14,654,991)</td>
<td>(13,251,836)</td>
</tr>
<tr>
<td>Female</td>
<td>-0.000</td>
<td>-0.018</td>
<td>-2,715,112**</td>
<td>2,791***</td>
</tr>
<tr>
<td></td>
<td>(0.208)</td>
<td>(0.212)</td>
<td>(50,570,958)</td>
<td>(33,933,829)</td>
</tr>
<tr>
<td>Household Size</td>
<td>-0.104</td>
<td>-0.160*</td>
<td>-24,140,997</td>
<td>-28,202,743**</td>
</tr>
<tr>
<td></td>
<td>(0.094)</td>
<td>(0.091)</td>
<td>(14,654,991)</td>
<td>(13,251,836)</td>
</tr>
<tr>
<td>Income</td>
<td>2.715***</td>
<td>2.791***</td>
<td>296</td>
<td>296</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.47</td>
<td>0.55</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td></td>
<td>0.42</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Notes: Dependent variable in columns (1) and (2) is ln(investible assets), and in columns (3) and (4) is investible assets. Columns (2) and (4) weight observations using study-specific post-stratification survey weights; columns (1) and (3) are unweighted. All specifications include state fixed effects. Robust standard errors in parentheses.

*p < 0.1; **p < 0.05; ***p < 0.01

4.2.3 Domain-Specific Predictions and Fingerprints and Stability

The domains were carefully designed not only for aggregate estimation of \( \alpha \), but also to test subtle and specific predictions of the model. The “exponential” domain is the simplest and allows for direct tests of our model to the Wagenaar and Sagaria (1975) model described in the introduction. Question 3 has no compounding and thus our model predicts that subjects should not make any error in contrast to the Wagenaar-Sagaria model. In our sample, 75.3% of subjects correctly stated the correct value of $104, and a further 4.0% answered with the interest-only value of $4. Another novel prediction of our model is that subjects will underestimate the value of a compounding asset if the interest rates are negative. Amongst the questions with negative interest rates (questions 4, 13, and 14), twice as many underestimate the asset as overestimate – another strong test favoring our model. Furthermore, 33% of all responses are exactly what a 0-Eddie would choose.

The fluctuating-interest domain and the portfolio domain both demonstrate people’s tendency
to take the arithmetic mean when combining multiple interest rates.\textsuperscript{30} A fully biased agent will use the arithmetic mean on both of these problems. Proposition 7 predicts that subjects will overweight the impact of the higher interest rate on the fluctuating-interest domain, but we also predict that they will underweight the impact of the higher interest rate in the portfolio domain. We find exactly this pattern amongst our subjects. For example, in question 7 of the fluctuating-interest domain, the mean growth is exactly zero, but the arithmetic mean of the interest rates is positive. On this question, 58\% of subjects believed the asset would increase while only 22\% believed it would decrease (and 20\% got it exactly right). Over the domain as a whole 17\% of responses left a full-bias fingerprint. In the portfolio domain, on question 24 for example, 74\% underestimated the impact of the high interest rate as predicted, and 15\% left a full-bias fingerprint. Over the domain as a whole 9\% left a full-bias fingerprint.

These two domains show how Eddie will incorrectly combine interest rates and as a consequence exhibit \textit{as-if risk preferences}. Keep in mind that there is no uncertainty in these problems. But an economist who thinks that uncertainty generates the fluctuating-interest and believes Eddie to be unbiased, would infer that Eddie is risk-seeking. In contrast an economist who thinks that Eddie’s portfolio faces risk would observe that Eddie is heavily invested in low-return assets. If the economist assumed a risk-return tradeoff, she would infer that Eddie is quite risk averse.\textsuperscript{31}

The catch-up savings and periodic savings domains agree with the directional predictions of the model, but the complexity of these questions implicates additional mathematical errors. Indeed, 33\% of responses in the periodic savings domain are below the sum of the contributions!

Fundamentally, the model presumes that Eddie treats assets separately rather than canceling common terms. For example, Question 8 (20 vs. 15 years at 13\%) can be reduced to solving for a principal of $100 growing for five periods at 13\% interest. More generally, since Eddie gets one round of interest exactly right, if he were to break down a problem into a sequence of iterated one-period problems, he would make no mistake. Question 10 in the catch-up savings domain was designed to directly address this issue; it asks what principal is needed for a one period delay in

\textsuperscript{30}The fluctuating-interest domain is the simplest example of an asset that has known but varying returns over time. The portfolio domain is the simplest example of a portfolio with two assets that have different returns.

\textsuperscript{31}Eddie’s as-if risk preferences and behavior under uncertainty is the subject of a sister paper in progress.
savings (9 periods vs 10 periods; the answer is \(\frac{1}{1+y_0}\)). Subjects do not seem to simplify this into a one period problem: 19% got the answer correct which is about their accuracy on other problems, and 67% respond with a principal in the biased direction.

To address the stability of our \(\alpha\) parameter estimates, we re-estimate equation (8) using only subjects’ responses to the second set of 10 questions and then compare our two estimates of \(\alpha\) within subjects. We are most interested in whether subjects identified as the “extreme” types – that is, with \(\alpha \in \{0, 1\}\) are consistent. This does appear to be the case. Of 126 control subjects identified as having \(\alpha = 0\) on the first set of questions, 78 (61.9%) yielded an estimate of \(\alpha = 0\) on the second set.\(^{32}\) In a linear probability model, we find that having \(\alpha = 0\) in the first set of questions raises the probability of having \(\alpha = 0\) in the second set by 22.28 percentage points (s.e. 4.41), while having initially had \(\alpha = 1\) lowers the probability by 41.61 percentage points (s.e. 9.21).

We also conclude that the bias is robust to the provision of information. This may be surprising, as the intervention made calculating the correct answer in the “exponential” domain all but trivial, but is not unreasonable – subjects were already free to use whatever tools they wanted to help them, including ones far more sophisticated than a simple graph. It is likely that subjects for whom the extra information would have been helpful were already using tools on their own. We find that subjects in the control and treated groups were statistically indistinguishable both in the pre-intervention and post-intervention phase. A Kolmogorov-Smirnov test of equal distributions in \(\alpha\) values calculated from pre-intervention data fails to reject at a significance of \(p=0.802\). The same test on \(\alpha\) values calculated from post-intervention data fails to reject at \(p=0.618\). Thus exponential-growth bias is unlikely to be eliminated by simple “nudges”: those who would benefit from the intervention may already be using tools of their own accord. More involved interventions may have some effect, however. For example, Song (2012) finds that face-to-face explanation of compounding along with financial advising has a large effect on savings in rural China.

\(^{32}\)We focus on control subjects to separate out any effect of the graphical intervention. It is not surprising, however, that a comparable 34 of 58 (58.6%) treated subjects identified as having zero in the first set of questions were also identified as having zero in the second set.
5 Conclusion

While the un-intuitively rapid growth of exponential functions has been observed for ages, the economic implications have only been considered recently. We derive and test the implications that consumers will make very specific – and very large – errors in their consumption plans, and the results of the first experiment overwhelmingly support this. Moreover, since the bias is fundamentally about the budget constraint, the model is modular and can thus be easily married to other economic settings or extensions. Moreover, the bias seems to prevail in the population as a whole and is a strong predictor of saving behavior even after controlling for the standard explanatory factors. The bias was robust to an intervention designed to make exponential growth more salient, and which could be used to obtain the correct response in some domains.

Ignoring the presence of EGB can potentially lead to substantially mis-specified econometric models. As a proof of concept, we generate simulated lifecycle consumption data assuming a constant interest rate of 5%, CRRA utility with EIS=2, full bias, no discounting, and mean-zero normally distributed error added to optimal consumption each period. We simulate a sample of 1000 agents randomly drawn between the ages of 19 and 66, observing their current and initial (age 18) consumption. This is as if the economist has two observations of each agent’s annual consumption, which can be used to infer the agent’s consumption growth over time. The economist knows the interest rate, knows that agents are homogenous, and knows that agents have a CRRA utility function, but does not know the EIS, δ, nor α. She can use non-linear least squares to estimate these three parameters. In Monte Carlo simulations, this unrestricted estimator performs well, estimating \( \hat{\text{EIS}}_U = 2.05 \) (s.e. 0.104), a discount factor \( \hat{\delta}_U = 0.97 \) (s.e. 0.03), and level of EGB \( \hat{\alpha}_U = -0.014 \) (s.e. 0.18). An economist who did not account for EGB will implicitly restrict \( \alpha = 1 \). The restricted estimator halves the EIS estimate to \( \hat{\text{EIS}}_R = 0.89 \) (s.e. 0.026) and imposes significant impatience in the discount factor \( \hat{\delta}_R = 0.68 \) (s.e. 0.025) to account for the biased agent’s lack of consumption growth over the lifecycle. While a simple exercise, this shows that EGB can have a large effect on estimates of important economic objects.

In principle, the re-framing of assets in the presence exponential-growth bias could actually
be used to identify the EIS. By reframing the period length at which interest compounds, one changes a biased agent’s perception of intertemporal prices. This affords the exogenous variation in intertemporal prices necessary to identify the EIS. The degree of additional structural assumptions required for identification depends on the quality of the data.

Ultimately field experiments with large-stake financial decisions are needed for ecological validity — since ultimately these are the target applications of interest. Additional research on the efficacy of interventions to combat EGB in the field is also necessary. While EGB was robust to the intervention in our Experiment 2, it is possible that other “nudges” designed around the predictions of our model could help improve welfare. The potential welfare consequences are quite large. Figure 6 plots the welfare costs of EGB as a function of $\alpha$ in a back-of-the-envelope calculation. For this exercise, we again consider CRRA utility and consumption that is dynamically re-optimized every year when the agent observes his true savings balance or debt. A representative agent earns the median US personal income between ages 20 and 65, and then retires until death at 78. We assume

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33 We thank an anonymous referee for this suggestion.

34 That is, while our Experiment 2 used a nationally representative sample, the data are still “artefactual”.

43
discounting is already reflected in interest rates, so the risk-free rate (post-retirement) is zero and risky rate (pre-retirement) is 5%. The welfare loss is measured as the fraction of additional income required to eliminate the loss of utility from EGB. As the figure shows, for a 0-Eddie the welfare effects are at least 2.8% of income, and can be as high as 5.2%.

Additional market implications of EGB also remain to be explored. Our results suggest that it is unlikely that competition would eliminate the bias; firms may find it more profitable to maximally exploit the bias instead (Gabaix and Laibson, 2006; Heidhues and Köszegi, 2010; Stango and Zinman, 2011). Consider a consumer’s choice of mutual funds as an illustration. Not only will managers have an incentive to present the arithmetic mean return of the fund (rather than the geometric mean), they may also take a riskier position in order to achieve a higher arithmetic mean. We expect that firms’ responses to exponential-growth-biased consumers will leave fingerprints in many such markets.

References


Lusardi, Annamaria and Olivia S. Mitchell, “How Ordinary Consumers Make Complex Economic


A General Model of Exponential-Growth Bias

We define a general *interest perception function* as the following:
**Interest Perception Function:** An interest perception function is a continuously differentiable function \( p : \mathbb{R}^T \times \{0, 1, \ldots, T\} \rightarrow \mathbb{R} \) such that:

1. **Boundary conditions** \( p(\vec{0}, t) = 1 \) for all \( t \) and \( p(\vec{i}, T) = 1 \) for all \( \vec{i} \)
2. **Compounding** \( \frac{\partial p(\vec{i}, t)}{\partial i_k} \geq 0 \) and \( \frac{\partial^2 p(\vec{i}, t)}{\partial i_j \partial i_k} \geq 0 \). For \( k \geq t \), if \( i_\tau = 0 \) for all \( \tau \neq k, \tau \geq t \), then \( \frac{\partial p(\vec{i}, t)}{\partial i_k} = 1 \)
3. **Irrelevancy of history** \( p(<i_1, \ldots, i_t, i_{t+1}, \ldots, i_{T-1}>, t) = p(<0, \ldots, 0, i_t, i_{t+1}, \ldots, i_{T-1}>, 0) \)
4. **Irrelevancy of order** If for some \( k, l \geq t, i_k = j_l, i_l = j_k \), and for any \( \tau \neq \{k, l\}, i_\tau = j_\tau \)

These assumptions define the minimum requirements for a well-behaved perception of interest rates. Assumption A1 simply states that an agent does not introduce discounting into a world with zero interest rates. Similarly, A2 states that agents correctly perceive that a single non-zero interest rate will discount period-T consumption on a one-for-one basis. This isolates the psychological phenomenon of neglecting compound growth from simple arithmetic mistakes. It also states that the cross-partials are weakly positive implying that the perception function is quasiconcave. A3 states both that historical rates do not directly affect beliefs about future growth, and that perceptions are not affected by prepending an arbitrary number of zero interest rate periods. This rules out, for instance, that a high interest rate experienced in an early period would raise the perceived growth rate in all subsequent periods. It also means that perceptions depend on the number of times interest compounds rather than the number of periods per se. Finally, assumption A4 states that any two future interest rates are interchangeable from the perspective of period \( t \). This is akin to a stationarity assumption, in as much as it implies all future periods are treated symmetrically.

**Exponential-Growth Bias:** An interest perception function \( p(\vec{i}, t) \) exhibits exponential-growth bias if:

\[
\frac{\partial}{\partial i_k} \left[ \frac{\partial p(\vec{i}, t)}{\partial i_j} \frac{\partial e(\vec{i}, t)}{\partial i_j} \right] \leq 0
\]

where \( e(\vec{i}, t) = \prod_{j=1}^{T-1} (1 + i_j) \) is the correct interest perception, \( T > 1 \), and the inequality is strict if \( j \neq k \). An interest perception function \( p \) exhibits greater exponential-growth bias than \( q \) if:

\[
\frac{\partial}{\partial i_k} \left[ \frac{\partial q(\vec{i}, t)}{\partial i_j} \frac{\partial e(\vec{i}, t)}{\partial i_j} \right] \leq 0
\]

where \( T > 1 \), and the inequality is strict if \( j \neq k \).

Equation (10) highlights the centrality of compounding to EGB. Combined with (A1)-(A2), it is clear that a biased agent will only misperceive the value of an asset with at least two periods of non-zero interest for it to grow. There is nothing to act on without any nonzero interest rates, and because there is no compounding with a single nonzero rate, (A2) implies correct perception even for a biased agent. Beginning with the addition of a second nonzero rate, however, the neglect of compounding implied by (10) implies that biased agents will begin to under-estimate exponential growth.
Moreover, because (A1) bounds the level of the perception function to be one when there is no interest, and (A2) requires that the partial derivative of the perception function with respect to an interest rate to be at least one and weakly increasing in all the other interest rates, (A1)-(A4) and (10) jointly imply that the interest perception function exhibiting the greatest extent of exponential-growth bias is one which fully linearizes growth: \( \hat{p}(\vec{i}, t) = 1 + \sum_{j=t}^{T-1} i_j \). This “fully biased” agent corresponds to the 0-Eddie in our parametric model.

Below, we present the generalized form of Lemmas 1 and 2 and Propositions ?? and 7. All other results hold for the general model as stated in the text. The main text explains the intuition and implications of these results.

**Lemma A.1 (Generalization of Lemma 1)** Suppose \( p \) and \( q \) both satisfy (A1)-(A4), and \( p \) exhibits greater exponential-growth bias than \( q \). Then:

\[
\frac{p(\vec{i}, t)}{p(\vec{i}, s)} \leq \frac{q(\vec{i}, t)}{q(\vec{i}, s)} \text{ for all } s > t.
\]

The inequality holds strictly if there exist \( j > k > t \) s.t. \( i_j > 0 \), \( i_k > 0 \).

For notational convenience, we define \( \hat{V}_{0,T}(\vec{y}, \vec{\tau}) \equiv \sum_{s=t}^{T} y_s \cdot \hat{p}(\vec{i}, s) \) as the perceived future value of an income stream and vector of interest rates.

**Lemma A.2 (Generalization of Lemma 2)** Given income streams \( \vec{y} \) and \( \vec{z} \) and interest rates \( \vec{\tau} \) such that:

\[\sum_{s=0}^{T} (\Pi_{j=s}^{T}(1+i_j)) y_s = \sum_{s=0}^{T} (\Pi_{j=s}^{T}(1+i_j)) z_s > 0\]

\[\sum_{s=0}^{T} (\Pi_{j=s}^{t}(1+i_j)) y_s > \sum_{s=0}^{T} (\Pi_{j=s}^{t}(1+i_j)) z_s \quad \forall t \in \{0, \ldots, T-2\},\]

then \( \hat{V}_{0,T}(\vec{y}, \vec{\tau}) < \hat{V}_{0,T}(\vec{z}, \vec{\tau}) \) for any interest perception function \( p \) that exhibits EGB.

**Proposition A.1 (Generalization of Proposition 5)** Let \( a(L, \vec{\tau}; p) \) s.t. \( \hat{V}(<L,-a,\ldots,-a>, \vec{\tau}; p) = 0 \) for an interest perception function \( p \). If \( p \) exhibits greater EGB than \( q \), then \(|a(L, \vec{\tau}; p)| < |a(L, \vec{\tau}; q)|\). Moreover, an agent with EGB will underestimate the number of periodic payments of any given size necessary to repay a given debt.

**Proposition A.2 (Generalization of Proposition 7)** Define \( m(\vec{\tau}; p) \) such that \( p(<m(\vec{\tau}; p), \ldots, m(\vec{\tau}; p), 0) = p(\vec{\tau}, 0) \) for all \( \vec{\tau} \in \mathbb{R}^T \). Then if \( p \) exhibits greater EGB than \( q \), \( m(\vec{\tau}; p) \geq m(\vec{\tau}; q) \). The inequality is strict if \( \exists j, k \) s.t. \( i_j \neq i_k \).

**B Proofs**

The proofs below use the notation of the general form of the model from Appendix A rather than the special case \( \alpha \)-Eddie model presented in the main text. Because the \( \alpha \)-Eddie model satisfies assumptions (A1)-(A4), these proofs imply the results in the main text directly.
Proof of Lemma A.1 (Generalization of Lemma 1) First, define $\tilde{\tau}(s) = < x, 0, ..., 0, i_{s+1}, ..., i_{T-1}, >$. By (A3), we can write $p(\tilde{i}, t) = p(\tilde{i}, t + 1) + \int_0^t \frac{\partial p(\tilde{i}, t, 0)}{\partial x} dx$ by using the the fundamental theorem of calculus. Continuing further:

\[
p(\tilde{i}, t) = p(\tilde{i}, T) + \sum_{j=0}^{T-t} \int_0^{t-j} \frac{\partial p(\tilde{i}, j, 0)}{\partial x} dx
\]

Thus

\[
\frac{p(\tilde{i}, t)}{q(\tilde{i}, t)} = 1 + \sum_{j=0}^{T-t} \int_0^{t-j} \frac{\partial p(\tilde{i}, j, 0)}{\partial x} dx
\]

(12)

Because $\tilde{\tau}(t)$ and $\tilde{\tau}(t+1)$ differ only on a single element, namely the (t+2)-th element $i_t$, we have by the definition of $p$ is more EGB than $q$ that

\[
\frac{\partial p(\tilde{\tau}(t), 0)}{\partial x} \leq \frac{\partial p(\tilde{\tau}(t+1), 0)}{\partial x}
\]

The ratio between the term in the numerator and the denominator in the two summations in (12) is decreasing in $j$. Reducing the summations an additional step will therefore lower the overall ratio, and thus we have:

\[
\frac{p(\tilde{i}, t)}{q(\tilde{i}, t)} = 1 + \sum_{j=0}^{T-t} \int_0^{t-j} \frac{\partial p(\tilde{i}, j, 0)}{\partial x} dx
\]

(13)

Rearranging (13) yields $\frac{p(\tilde{i}, t)}{p(\tilde{i}, t+1)} \leq q(\tilde{i}, t+1)$, which can be iterated as many times as needed to yield $\frac{p(\tilde{i}, t)}{p(\tilde{i}, t+n)} \leq q(\tilde{i}, t+n)$ for any positive integer $n$.

Proof of Lemma A.2 (Generalization of Lemma 2) In this and all subsequent proofs, let the period-t perception of final wealth net of obligations incurred in periods $\{0, \ldots, t-1\}$ be given by $\tilde{W}_{i, T}(\tilde{y}) = \left[ \sum_{s=1}^{t-1} (y_s - c_s) \prod_{j=1}^{s-1} (1 + j) \right] p(i, t) + \sum_{s=t}^T p(i, s) y_s$, and note that $\tilde{W}_{i, T}(y) = \tilde{V}_0, T(\tilde{y}, \tilde{t})$.

By (ii), $y_0 > 0$. Define a wealth-preserving shift of income from period zero to period one $\tilde{y}^{(1)} = \tilde{y} + (< z_0 - y_0), (y_0 - z_0)(1 + i_0), 0, \ldots, 0 >$. Note that $(1 + i_0)p(i, s + 1) > p(i, s)$ by Lemma A.1, since $p$ exhibits EGB.

Then $\tilde{W}_{i, T}(\tilde{y}^{(1)}) - \tilde{W}_{i, T}(\tilde{y}) = ((1 + i_0)p(\tilde{i}, 1) - p(\tilde{i}, 0))(y_0 - z_0) > 0$.

Similarly, for $s = 2, \ldots, T$, recursively define $\tilde{y}^{(s)} = \tilde{y}^{(s-1)} + < 0, \ldots, (b_s - a_s^{(s-1)}), (a_s^{(s-1)} - b_s)(1 + i_{s-1}), 0 >$, that is, by shifting $(y_s - z_s)$ from period $s-1$ to period $s$, at the interest rate $i_{s-1}$. By (iii), $(y_s - z_s) > 0$, and so $\tilde{W}_{i, T}(\tilde{y}^{(s-1)}) - \tilde{W}_{i, T}(\tilde{y}^{(s)}) = \left[ p(\tilde{i}, s)(1 + i_{s-1}) - p(\tilde{i}, s - 1) \right] (y_{s-1} - z_{s-1})$ for all $s < T$, and equal to zero for $s = T$. From (i), however, we have that $\tilde{y}^{(T)} = \tilde{z}$. Thus $\tilde{W}_{i, T}(\tilde{y}) < \tilde{W}_{i, T}(\tilde{y}^{(1)}) < \ldots < \tilde{W}_{i, T}(\tilde{y}^{(T-1)}) = \tilde{W}_{i, T}(\tilde{y}^{(T)}) = \tilde{W}_{i, T}(\tilde{z})$.

Proof of Proposition 1 We can rewrite equation (6), using the general perception function and separating period-0 consumption, as $c_0 p(\tilde{t}, 0) + g(c_0) = \tilde{W}_{i, T}(\tilde{y})$, where

\[
g(c_0) = \sum_{s=1}^T p(\tilde{i}, s) u^{-1} \left( \frac{p(\tilde{i}, s) u'(c_0)}{p(\tilde{i}, 0) \delta^s} \right)
\]

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\[ \frac{\partial g(c_0)}{\partial c_0} > 0 \] because utility is concave and increasing. Now suppose we reduce period-s income by \( \epsilon \) and increase period \((s+1)\) income by \((1+i_s)\)\( \epsilon \). By Lemma 2, \( \Delta \tilde{W}_{0,T}(\tilde{y}) > 0 \) if \( s < T-1 \), and \( \Delta \tilde{W}_{0,T}(\tilde{y}) = 0 \) if \( s = T-1 \). Thus \( c_0 \) strictly increases for \( s < T-1 \) and is unchanged for \( s = T-1 \).

**Proof of Proposition 2** Consider a perception function \( p(\tilde{i}, 0) \) and a less biased perception function \( q(\tilde{i}, 0) \). We apply equation (6) to both perception functions. The RHS is equal for both because all wealth is received lump sum at \( T \) and so there is no misperception about the wealth available in period \( T \) units.

\[
\hat{c}_0 p(\tilde{i}, 0) + \left[ \sum_{s=1}^{T} p(\tilde{i}, s) u' \left( \frac{p(\tilde{i}, s) u'(c_0)}{p(\tilde{i}, 0) \delta^s} \right) \right] = \hat{c}_0 q(\tilde{i}, 0) + \left[ \sum_{s=1}^{T} q(\tilde{i}, s) u' \left( \frac{q(\tilde{i}, s) u'(c_0^q)}{q(\tilde{i}, 0) \delta^s} \right) \right]
\]

where \( c_j^q \) represents consumption with the perception function \( j \). By Lemma A.1 \( \frac{p(\tilde{i}, s)}{p(\tilde{i}, 0) \delta^s} > \frac{q(\tilde{i}, s)}{q(\tilde{i}, 0) \delta^s} \).

By (A1), (A2), and the definition of EGB \( p(\tilde{i}, t) \leq q(\tilde{i}, t) \). If \( c_0^p \leq \hat{c}_0^q \) then the inverse-utility term on the LHS is less than the inverse utility term on the RHS since \( u'(\cdot) \) and \( u'^{-1}(\cdot) \) are decreasing functions. But this is a contradiction since every term on the LHS is now less than every term on the RHS. Therefore \( c_0^p > \hat{c}_0^q \).

**Proof of Proposition 3** By Proposition 1 it is sufficient to show that an agent will over-consume when all their income is included in their period-0 endowment; any deferment will exacerbate the over-consumption so long as all cash flows are weakly positive. Let \( e(\tilde{i}, t) \) denote exponential perception. We prove by contradiction.

Suppose \( c_0 \leq \hat{c}_0 \) and \( -\frac{u'(c)}{u''(c)c} > 1 \). Then from the agent’s Euler condition,

\[
u'(\hat{c}_s) \frac{p(\tilde{i}, 0) \delta^s}{p(\tilde{i}, s)} = u'(c_0) \geq u'(\hat{c}_s) = u'(\hat{c}_s) \frac{e(\tilde{i}, 0) \delta^s}{e(\tilde{i}, s)}
\]

(14) By Lemma A.1, \( \frac{p(\tilde{i}, 0)}{p(\tilde{i}, s)} < \frac{e(\tilde{i}, 0)}{e(\tilde{i}, s)} \Rightarrow \hat{c}_s < c_s^* \)

Now we consider two cases. Case 1: \( \hat{c}_s p(\tilde{i}, 0) \geq c_s^* \). Multiply the LHS of this inequality with the LHS of (14) and the RHS of the inequality with the RHS of (14). This yields \( \hat{c}_s u'(\hat{c}_s) \geq c_s^* u'(c_s^*) \). Observe \( c \cdot u'(c) \) is an increasing function iff \( \frac{d}{dc}(c \cdot u'(c)) > 0 \Leftrightarrow -\frac{u'(c)}{u''(c)c} > 1 \) which is the assumed EIS condition. Therefore \( \hat{c}_s \geq c_s^* \) This is a contradiction, as \( \hat{c}_s < c_s^* \).

Case 2: \( \hat{c}_s p(\tilde{i}, 0) < c_s^* p(\tilde{i}, 0) \). The budget constraint then implies:

\[ c_0 + \sum_{s=1}^{T} \frac{p(\tilde{i}, s)}{p(\tilde{i}, 0)} \hat{c}_s < c_0 + \sum_{s=1}^{T} \frac{e(\tilde{i}, s)}{e(\tilde{i}, 0)} c_s^* = y_0 \]

Which is a violation of Walras’ law (not expending the full budget), and therefore \( c_0 \) cannot be optimal. Thus \( c_0 > c_0^0 \).
Proof of Proposition 4 Let \(\hat{c}_{t,T}\) denote the agent’s period-\(t\) expectation of consumption in period \(\tau > t\), and let the period-\(t\) perception of final wealth net of obligations incurred in periods \(\{0, \ldots, t-1\}\) be given by \(\hat{W}_{t,T}(\vec{y})\) as in the proof of Lemma 2.

We note that consumption in every period is a normal good, and from the perceived budget constraint \(\hat{c}_{t,T}p(\vec{t}, t) + \sum_{s=t+1}^{T} p(\vec{i}, s)\hat{c}_{t,s} = \hat{W}_{t,T}\), it is clear that if \(\hat{W}_{t,T}\) increases, \(\hat{c}_{t,s}\) must also increase for all \(s \in \{t, \ldots T\}\).

At time \(t\), the budget constraint yields \(\sum_{s=t+1}^{T} \hat{c}_{t,s}p(\vec{i}, s) = \hat{W}_{t,T} - c_{t}p(\vec{i}, t)\)

At time \(t+1\),

\[
\hat{W}_{t+1,T} = \left[\sum_{s=0}^{t-1}(y_{s} - c_{s}) \prod_{j=s}^{t-1}(1 + i_{j})\right] p(\vec{i}, t+1)(1 + i_{t}) + \left(\sum_{s=t+1}^{T} p(\vec{i}, s)y_{s}\right) + (y_{t} - c_{t})p(\vec{i}, t + 1)(1 + i_{t})
\]

and the budget constraint yields \(\sum_{s=t+1}^{T} \hat{c}_{t+1,s}p(\vec{i}, s) = \hat{W}_{t+1,T}\). Thus the perceived budget will decrease only if \(\hat{W}_{t,T} - c_{t}p(\vec{i}, t) > \hat{W}_{t+1,T}\), i.e.:

\[
\left[\sum_{s=0}^{t-1}(y_{s} - c_{s}) \prod_{j=s}^{t-1}(1 + i_{j})\right] p(\vec{i}, t) - (1 + i_{t})p(\vec{i}, t + 1) > 0
\]

\[
\sum_{s=0}^{t}(y_{s} - c_{s})\prod_{j=s}^{t-1}(1 + i_{j}) < 0
\]

since \(p(\vec{i}, t) < (1 + i_{t})p(\vec{i}, t + 1)\) by Lemma 1 since \(p\) exhibits EGB. Thus \(\sum_{s=t+1}^{T} \hat{c}_{t+1,s}p(\vec{i}, s) < \sum_{s=t+1}^{T} \hat{c}_{t,s}p(\vec{i}, s)\), and from the Euler equation each term in the sequence \(c_{t+1,s} < c_{t,s}\) giving the desired result.

Proof of Proposition A.1 (Generalization of Proposition 5) By definition of \(a(\cdot)\),

\[
a(L, \vec{v}; p) \sum_{k=1}^{T} \frac{p(\vec{v}, k)}{p(\vec{v}, 0)} = L = a(L, \vec{v}; q) \sum_{k=1}^{T} \frac{q(\vec{v}, k)}{q(\vec{v}, 0)}
\]

By Lemma A.1, \(\frac{p(\vec{v}, k)}{p(\vec{v}, 0)} > \frac{q(\vec{v}, k)}{q(\vec{v}, 0)}\) for all \(k \leq T - 2\) (and equal for \(k = T - 1\)). Thus the summation in the leftmost term above is strictly greater than the summation in the rightmost term. For the equality to hold, therefore, we require \(a(L, \vec{v}; p) < a(L, \vec{v}; q)\).

Similarly, letting \(e(\cdot)\) denote correct exponential perception,

\[
L = a(L, \vec{v}; e) \sum_{k=1}^{T} \frac{e(\vec{v}, k)}{e(\vec{v}, 0)} > a(L, \vec{v}; p) \sum_{k=1}^{T} \frac{e(\vec{v}, k)}{e(\vec{v}, 0)}
\]

An agent expecting under \(p\) to pay off a loan of \(L\) in \(T\) periods will thus have a strictly negative balance in period \(T\).
Proof of Proposition 6 The right-most inequality is an immediate result:

\[
\hat{V}_{0,n}(\bar{y}^{(n)}, \bar{i}^{(n)}; \alpha) = y_0 \cdot \left( \prod_{s=1}^{n} (1 + \alpha i_s) + \sum_{s=1}^{n} (1 - \alpha) i_n \right) < y_0 \cdot \left( \prod_{s=1}^{n} (1 + i_n) \right) = y_0 \cdot (1 + i_0) = \hat{V}_{0,n}(\bar{y}^{(n)}, \bar{i}^{(n)}; 1)
\]  

(15)  

(16)

The other inequalities also result from straightforward algebra:

\[
\hat{V}_{0,n}(\bar{y}^{(n)}, \bar{i}^{(n)}; \alpha) > y_0 \cdot \left( 1 + \sum_{s=1}^{n} i_n \right) = y_0 \cdot \left[ 1 + n \left( (1 + i_0)^{1/n} - 1 \right) \right]
\]

(17)

We first note that the right-most term in (17) is a decreasing function in \(n\) for \(n > 1\), as:

\[
d/dn \left[ 1 + n \left( (1 + i_0)^{1/n} - 1 \right) \right] = (1 + i_0)^{1/n} \left( 1 - \frac{\ln(1 + i_0)}{n} \right) - 1 < 0
\]

(18)

since \((1 - \frac{\ln(1 + i)}{n})^n < \frac{1}{1+i} \) for \(i > 0, n > 1\) We thus take the limit to find a lower bound, and find:

\[
\lim_{n \to \infty} y_0 \left[ 1 + n \left( (1 + i_0)^{1/n} - 1 \right) \right] = y_0 \left[ 1 + \ln(1 + i_0) \right] = \hat{V}_{0,1}(\bar{y}, <ln(1 + i_0)>; \alpha) > y_0
\]

(19)

Proof of Proposition A.2 (Generalization of Proposition 7) Consider a contour of the perception function \(p(\cdot, 0) = p(\bar{i}, 0)\) in interest-rate space \(\mathbb{R}^{T-1}\), i.e. the set \(\{ \bar{j} \in \mathbb{R}^{T-1} | p(\bar{j}, 0) = p(\bar{i}, 0) \}\). By construction, \(p(\bar{m}, 0) \equiv p(\bar{i}, 0)\). Along the contour \(\sum_{s=0}^{T-1} \frac{\partial p(\bar{j},0)}{\partial \bar{i}_s} = 0\). From (A4) the contour is symmetric about the 45-degree line, that is it is symmetric about a ray from the origin defined by \(\gamma \bar{I}\) where \(\gamma > 0\). From (A2) the perception function is quasiconcave in \(i_t\) since the function is strictly increasing in \(i_t\) and the \(\frac{\partial^2 p(\bar{i},0)}{\partial i_j \partial i_k} \geq 0\) for all \(j, k\). This implies that the contours of the perception function are bowed inward toward the origin.

The gradient of the perception function \(\nabla_i p(\bar{i}, 0)\) is orthogonal to the contour and hence fully determines the contour’s shape (see Figure B.1). By definition the intersection between the contour and \(\gamma \bar{I}\) uniquely determines \(m(\bar{i}; p) = \gamma\). We now wish to show that the contour of \(p(\bar{i}, 0) = c\) must intersect the ray at a higher \(\gamma\) then \(q(\bar{i}, 0) = k\). By (A4), \(\frac{\partial p(\gamma \bar{I},0) / \partial i_j}{\partial p(\gamma \bar{I},0) / \partial i_k} = \frac{\partial q(\gamma \bar{I},0) / \partial i_j}{\partial q(\gamma \bar{I},0) / \partial i_k} = 1\). For \(i_j < i_k\), because \(p\) is more biased than \(q\), \(-\frac{\partial q(\bar{i},0) / \partial i_j}{\partial q(\bar{i},0) / \partial i_k} < -\frac{\partial p(\bar{i},0) / \partial i_j}{\partial p(\bar{i},0) / \partial i_k}\). This expression is analogous to a comparison of marginal rates of substitution. This implies that at all points at \(\bar{i}, \nabla_i q(\bar{i}, 0)\) will be directed more towards \(\gamma \bar{I}\) than is \(\nabla_i p(\bar{i}, 0)\), and the contour \(q(\bar{i}, 0) = k\) will curve more steeply towards the origin, and as a consequence \(m(\bar{i}; q) < m(\bar{i}; p)\).
Table C.1 displays the results of a linear regression of $\alpha$ on several covariates. The model predicts that $\alpha$ should be positively associated with bankruptcy, payday loan use, balance on credit cards, no financial advice, and negatively associated with the use of tools and advice. The theory is silent on the other covariates.

We find that the use of a financial calculator is substantially and significantly associated with greater accuracy. We find no evidence for other associations. This certainly in part reflects the limited sample size. Our sample only has 36 subjects who use payday loans and only 38 people who have gone bankrupt. Future research should explore these relationships where high-cost debt users are oversampled. We find no association with education and $\alpha$. Other model specifications that use categories for education (< HS, HS, some college, college) decade-length age categories, and a dummy to separate larger households from smaller household generate similar non-significant results. This is consistent with the findings of Cronqvist and Siegel (2012) who find that general education explains a minuscule fraction of the variation in financial biases. It is also consistent with a casual observation of the first table Almenberg and Gerdes (2012), in which there is no clear pattern between $\alpha$ as a function of education or income. In contrast Stango and Zinman (2009) find that bias is negatively correlated with income, education, and being male. Their sample size is about six times larger so this could explain the discrepancy between our results.
Table C.1: Correlates of Alpha

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* p < 0.1; ** p < 0.05; *** p < 0.01

Ordinary least squares, the dependent variable is \( \alpha \). The omitted religious group is protestant. Specifications (2)–(8) include State fixed effects.
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Figure C.2: Example of Static Task for Experiment 1

Notes: Subjects were given a task to maximize an explicitly stated utility function subject to the intertemporal budget constraint corresponding to an assigned income vector and interest rate vector. Savings and debts would accrue interest as a function of the interest rates. Earnings were strictly increasing in the achieved utility. Subjects in the Static arm were required to state a comprehensive plan for all periods simultaneously. To aid comprehension, the task was framed as a game to feed a “digital dog”. Subjects were allotted “bucks” (income) on various “days” (periods) which could be used to purchase “dog food” (consumption). The dog food generated “wags” of the tail (utils). The object to be maximized was the sum of “tail wags” (utils) over the course of all the days.
Figure C.3: Example of Dynamic Task for Experiment 1

Notes: Subjects were given a task to maximize an explicitly stated utility function subject to the intertemporal budget constraint corresponding to an assigned income vector and interest rate vector. Savings and debts would accrue interest as a function of the interest rates. Earnings were strictly increasing in the achieved utility. Subjects in the Dynamic arm stated consumption iteratively for one period at a time, and the new balance would be displayed before subjects stated consumption for the next period. To aid comprehension, the task was framed as a game to feed a “digital dog”. Subjects were allotted “bucks” (income) on various “days” (periods) which could be used to purchase “dog food” (consumption). The dog food generated “wags” of the tail (utils). The object to be maximized was the sum of “tail wags” (utils) over the course of all the days.
Figure C.4: Example of Task for Experiment 2 with Graphical Intervention

Notes: Subjects estimate the value of X to make both assets equal. For the control group and the first 10 questions asked of the treated group, there was no graph, but the presentation was otherwise identical. The latter 16 questions asked of the treated group were accompanied by the graph. The graph was interactive allowing users to observe the growth over different time horizons.